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Deliverable 1.2

Report on learning tasks and cognitive models

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1.5	8/10/14	Section 4.3 developed
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Executive Summary

Task design is a challenging task. In iTalk2Learn, the combination of tasks to encourage procedural and conceptual knowledge (structured and exploratory tasks) renders task design even more challenging. This deliverable reports on the design of exploratory tasks used in the learning platform and how structured content (from Maths-Whizz and Fractions Tutor) is being interweaved with appropriate exploratory activities (T1.2). We also report on the related project task that involves the identification and operationalisation of relevant topics, learning objectives and problem solving strategies for elementary mathematics (T1.3).

Fractions are notoriously difficult to teach and learn. With respect to the identification of learning objectives and their operationalisation (T1.3), Section 2 presents in detail how we researched and created an original coherent system for fractions learning that we are using to support students develop robust fractions knowledge. How this coherent system is being shared and published for teachers in various forms is identified in D6.3.2.

Section 3 discusses the role that errors and misconceptions play in learning and teaching mathematics and fractions in particular; and Appendix I provides the typical misconceptions related to fractions we are drawing upon in our task design. We make the original distinction between a) 'global fraction misconceptions' that seem to be endemic in students' understanding of fractions and can be seen in most situations related to fractions and b) 'situated fraction misconceptions' that manifest themselves in specific combinations of task content, task context and representations. The misconceptions inform the design of task-dependent and task-independent support to address them. With respect to content (T1.2), the deliverable also presents our analysis of the state-of-the-art of teaching fractions in the field of mathematics education and how it has influenced our task design. The unique framework we are using for task design is explained. Using this ensures that a variety of tasks for robust elementary mathematics learning are incorporated into the platform.

Section 4 presents our approach to exploratory task design and structured task selection. It explains how we selected the structured tasks (i.e. Whizz and Fractions Tutor tasks) to interleave with the specific Fractions Lab exploratory tasks and the principles that guided our decisions. As WP5 reports in more detail in D5.2, teachers and students have been involved explicitly in this work and have influenced task design and re-design. The selected tasks are reported in Appendix III (structured) and IV (exploratory). This information contributes to WP2 (D2.2.2) work and in both the sequencer, as a way to ameliorate the performance prediction, and the task-dependent support, as a way to inform decision making with respect to feedback generation. In addition, the results here inform WP3 (D3.3.1) for the common mathematical terms and words that can be used by the speech recognition system and the task-independent support components.

This information and the task-design work overall also contributes to milestone M24 (Pedagogical Interventions) as they play a role in the definition of the intervention model for each task in D1.3. While officially the task-design work concludes with this deliverable, task-related aspects will still concern the project as tasks are aligned with parallel work in WP2, WP3 and WP4 particularly with respect to any technical representation needs and metadata of the content.

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List of Abbreviations

UHi	Universitat Hildesheim
IOE	Institute of Education, University of London
TL	Testaluna SRL
RUB	Ruhr-Universität Bochum
BBK	Birkbeck College – University of London
Whizz	WHIZZ Education Limited
SAIL	SAIL Labs Technology AG
ELE	Exploratory Learning Environment
ITS	Intelligent Tutoring System
WP	Work Package
FT	Fractions Tutor
ELE	Exploratory Learning Environment
GM	Global Misconception
SM	Situated Misconception
CGG	Coarse-grain goal
FGG	Fine-grain goal
TDS	Task-dependent support

1 General Introduction

This deliverable reports on the design of the exploratory tasks that are used in the learning platform and explains how existing structured content (from Maths-Whizz and Fractions Tutor) are being reused and interweaved with appropriate exploratory activities (T1.2). It also reports on the task of identifying and operationalising relevant topics, learning objectives and problem solving strategies for elementary mathematics (T1.3).

The deliverable is structured as follows. The next sub-section presents the relationship of the deliverable with the project and highlights the innovations behind tasks T1.2 and T1.3. Section 2 sets the scene for how we are developing an original coherent system for fractions learning that we are using to support students to develop robust fractions knowledge within the platform. Section 3 explains the importance of identifying and addressing misconceptions in learning. Section 4 identifies how the structured and exploratory tasks have been selected and/or designed for the platform. Lastly, Section 5 presents the summary and implications for the project.

1.1 Relationship to the project and innovations

In reference to the iTalk2Learn objectives, WP1 in general aims to provide the pedagogical background and content required in the project with respect to learning processes and possible guidance or support required in elementary fractions, the domain chosen by the project. As mentioned in other deliverables, the project selected fractions (in particular, fractions equivalence, addition and subtraction) as the target domain because of the widely acknowledged difficulty that students have in learning fractions and the richness fractions afford with respect to different representations and interpretations (Charalambous & Pitta-Pantazi, 2007). Furthermore, Siegler *et al*. (2012) found that elementary students' knowledge of fractions and division at 10 years of age is a uniquely accurate predictor of their attainment in algebra and overall maths performance five or six years later.

The iTalk2Learn work packages:

WP number	WP name	Lead beneficiary
1	Robust Learning in Elementary Mathematics	IOE
2	Adaptive Intelligence for Robust Learning Support	UHi
3	Intuitive Interaction Interfaces for Elementary Mathematics	TL/SAIL
4	Deployment and Integration	BBK
5	Data Collection and Evaluation	RUB
6	Dissemination and Exploitation	Whizz
7	Project Management	UHi

The WP1 work reported upon in this deliverable contributes directly to all objectives of the project in the following ways:

Objective 1 *Provide an open-source platform for intelligent support systems integrating structured practice and exploratory, conceptually-oriented learning;*

- Development of the conceptually-orientated contents of the open-source platform (WP4);
- Input to the representation of tasks (task-information package) (WP4)

Objective 2 *Provide state-of-the-art and highly innovative reference implementations of plugins for the platform that could be used in a wide range of application domains*;

- Input to task-dependent and task-independent support (WP2) through specification of the coarse- and fine-grain goals and errors and misconceptions;
- The work reported here was a prerequisite of the work in the intervention model reported in D1.3 and eventually implemented as a switching and sequencing engine (WP2)

Objective 3 *Promote our understanding of the role of the different modalities of speech and direct manipulation of multiple or alternative representations in learning elementary mathematics through digital technologies*;

- Providing mathematical vocabulary and indicative utterances which influence the design of the speech recognition vocabulary and language model and supports evaluations regarding precision and recall (WP3)
- Considering how best to employ multiple representations in learning elementary mathematics using digital technologies (WP3);

Objective 4 *A summative evaluation of activities and support features generated by our intelligent learning support platform*;

- The formative evaluations in WP5 of the work in WP1 (reported in D5.2) inform both the development of D1.3 and the summative evaluation in WP5.

With respect to innovation, the coherent system for learning fractions using interpretations, representations, fraction types, fine-grain goals and task types that is described in Section 2 is in itself a contribution in the field of Mathematics Education and as such we have begun to disseminate this and considering possible future exploitations (e.g. for teacher professional development). In addition, the five fraction dimensions will be operationalised within the platform to help the system provide task-dependent and task-independent support as well as sequencing and switching of the tasks (see D4.2.1). Finally, the work described here contributes to milestone M24 (Pedagogical Interventions) as it plays a role in the definition of the intervention model for each task in D1.3.

1.2 Published results and other indicators of impact

We have published and disseminated our pedagogical subject content work within the mathematics education and elementary teaching communities. In relation to this deliverable and WP1 in particular, the two areas we have focused upon in Year 2 have been (a) the iTalk2Learn Matrix which was first introduced in the Year 1 annual report and later in the M18 version of this deliverable and (b) students' errors and misconceptions in fractions. In addition the work here has enabled us not only to continue informing the design of Fractions Lab but (c) evaluate the impact Fractions Lab and the exploratory tasks have on students' conceptual knowledge.

<u>The iTalk2Learn Matrix</u>

The Matrix (see Appendix I) has been warmly welcomed by the mathematics education community as it has been shared in conference papers, workshops and key note presentations. This innovative way of presenting the relationships between the interpretations and representations is previously unpublished and therefore presents a new way of thinking about fractions in mathematics education.

- Hansen, A. & Leeming, J. (2014a) *Fractions, decimals, percentages, ratio and proportion.* In Witt, M. (ed) (2014) Primary mathematics for trainee teachers. London: Learning Matters/SAGE.
- Hansen, A. (2014) *Errors and misconceptions in mathematics - a focus on fractions* Plenary address at Edge Hill University's Day Meetings for Mathematics Specialist Teachers (130 teachers in Central England: 7/3/14 and 10/5/14; 140 teachers in Northwest England: 14/3/14 and 17/5/14).
- Hansen, A. (2014) *Teaching fractions* Keynote address at the Shropshire Primary Mathematics Conference 17/9/14 for 85 teachers.

<u>Errors and misconceptions</u>

- Hansen, A. (2014b) *Number: fractions, decimals and percentages.* In Hansen, A. (ed) Children's errors in mathematics (3rd edition). London: Learning Matters/SAGE. (Available from http://www.italk2learn.eu/wp-content/uploads/2014/09/Childrens_Errors_in_Maths_3rd_Ed.pdf).

<u>Fractions Lab evaluation</u>

- Hansen, A., Geraniou, E. & Mavrikis, M. (2014) *Designing interactive representations for learning fractions* Presentation at the British Educational Research Association Conference, London 23-25 September, 2014. (Slides available on: http://www.slideshare.net/italk2learn/designing-interactive-representations-for-learning-fractions)

2 Developing a coherent system for fractions learning

iTalk2Learn aims at helping elementary students develop robust knowledge in the field of fractions in general, and equivalence, addition and subtraction of fractions in particular. We focus on this aspect of mathematics because fractions performance predicts students' mathematics achievement in high school, above and beyond the contributions of whole number arithmetic knowledge, verbal and non-verbal IQ, working memory, and family education and income (Siegler et al, 2012), yet fractions are one of the most difficult aspect of mathematics to teach and learn (Charalambous & Pitta-Pantazi, 2007). The difficulty arises due to the significant complexity of fractions. In D1.1 we reported the number of ways fractions can be interpreted and the number of (graphical) representations teachers can draw upon to teach fraction which is just one aspect of why fractions are complex. During Year 2 of the project we developed our thinking significantly regarding what constitutes a coherent system for fractions learning to include coarse-grain goals, fine-grain goals, different types of fractions (e.g. unit, proper, improper) and types of tasks. This section outlines how coarse-grain goals and the five dimensions form the iTalk2Learn's consortiums interpretations of what a coherent system for fractions learning looks like. This coherent system for fractions learning is revisited in Section 4 of this deliverable where its use by other WPs is explained more fully.

What follows are two sections that address the coherent system for fractions learning. We see them as two sides of the same coin. The first addresses a macro-level system, looking at the coarse-grain curricula approach and the second takes a micro-level approach to identify what is required for students to learn fractions on the individual task-level, introducing the 'five dimensions of fractions learning'.

2.1 Coarse-grain goals, a macro-level approach

When developing a coherent system for fractions learning we must firstly be mindful of the learning trajectory that students will typically follow demonstrating a more sophisticated understanding of fractions as they progress. By its very nature it is hierarchical and indeed we are reminded "curriculum development and instruction must consider hierarchy" (NCTM Curriculum and Evaluation Standards for School Mathematics. 1989:48).

To formalise this learning trajectory we identified a set of learning objectives referred to in the project as coarse-grain goals (CGGs) that represent the steps students typically follow in order to develop a robust knowledge of adding and subtracting fractions and their prerequisites (e.g., making fractions equivalent). We took into account mathematics curricula, research literature, advice from mathematics education experts and project partners' experience teaching students fractions. The CGGs are presented in Table 1 and a justification for the order of the goals and the inclusion of each is provided below.

Table 1: Coarse-grain goals

No.	Coarse-grain goal	
0	Familiarization	
1	Fractions as part of a whole	
2	Equivalent fractions	
3	Add and subtract two fractions	
	3a+	Add two fractions with the same denominator
	3a-	Subtract two fractions with the same denominator
	3b+	Add two fractions with denominators that are multiples of the same number
	3b-	Subtract two fractions with denominators that are multiples of the same number
	3c+	Add two fractions with unlike denominators
	3c-	Subtract two fractions with unlike denominators

2.1.1 Familiarisation

Although not a fractions-related learning objective, this goal has been included to familiarise students with the platform's user interface, content (i.e. exploratory learning environment and structured tasks) and functionality (e.g. moving to the next task, asking help) to enable students to effectively work through the iTalk2Learn platform.

2.1.2 Fractions as part of a whole

Understanding that fractions are part of a whole is crucial for students to develop rational number understanding because it is fundamental to all interpretations of fractions as well as being an important language-generating construct (Kieren, 1981) and yet through our own analyses of textbooks (see D1.1), discussions with teachers (see D6.3.2) and mathematics education experts (e.g. Professor John Mason, Professor Anne Watson and the IOE Mathematics Education Special Interest Group) we see this is often overlooked by teachers and instructional resources. This rational number sense helps them to understand how fractions are different to whole numbers and helps them to conceptualise equivalence, addition and subtraction of fractions. Understanding that fractions are part of a whole also begins to address the global fraction misconception of treating numerators and denominators as whole numbers. Learning to add and subtract fractions without understanding what is being added is futile and inhibits conceptual understanding (see Section 3.6 for further discussion on global fraction misconceptions and how misconceptions relate to conceptual development).

2.1.3 Equivalent fractions

Equivalence is a pre-requisite for comparing fractions and operating with fractions (English & Halford, 1995; Ni, 2001; Pantziara & Philippou, 2012; Wong and Evans, 2007). Fraction equivalence constitutes one of the most important mathematical ideas in the primary school and a major difficulty for students, due to its multiplicative nature (Ni, 2001). Charalambous and Pitta-Pantazi (2007) argue that instead of using "difficult" models with students, teachers should help them "master other notions, such as the equivalence". Although some (e.g. Wong & Evans, 2007) position fraction equivalence as "one concept within the extensive fraction schemata" we argue that equivalence is omnipresent because it is not possible to operate with most fractions without using equivalence. Therefore we felt that it was important that equivalence was embedded for both purpose and utility (Ainley, Pratt and Hansen, 2008) within later tasks (i.e. coarse grain goals 3b and 3c), as well as a stand-alone coarse-grain goal.

2.1.4 Add and subtract fractions

The area of addition and subtraction has been studied less extensively than multiplication and division of fractions (Verschaffel, Greer, & Torbeyns, 2006, p. 65, cited by Charalambous, Delaney, Hsu & Mesa, 2010). Like other aspects of fractions, adding and subtracting fractions is difficult to learn because children tend to use the additive structures of whole numbers (Lamon, 2012; Newstead & Murray, 1998) and as a result make systematic errors (Vinner, Hershkowitz & Bruckheimer, 1981).

Coarse Goal 3 is the final goal, but it is split into six sub-goals, reflecting the complexity of learning to add and subtract fractions. The subdivisions have been made based on curricula from around the world (*c.f.* Department for Education, 2013; Ministerium für Schule und Weiterbildung des Landes Nordrhein-Westfalen, 2007a, 2007b) and on our own experience of how students learn fractions. They reflect the different strategies that students need to master to be competent with adding and subtracting fractions and they are listed here reflecting a typical learning hierarchy so students can be most appropriately scaffolded through the stages. To avoid repetition in the discussion below, addition and subtraction have been combined. However, in the platform itself the two have been separated so that students can develop their understanding about these different but complementary mathematical ideas independently of each other in the exploratory and structured tasks. Interleaving structured tasks will bring addition and subtraction together with the intention that students will see the interrelated nature of the two. This draws upon the work of Hansen (2008) who introduces the notion of task efficiency drive where students begin their learning journey in a task or series of tasks with an intuitive or ad hoc notion of the concepts, they develop situated abstractions within the task(s) and emerge with understanding of the relationships intentionally designed into the task(s).

Adding and subtracting two fractions with the same denominator (3a) is the simplest because there is no need to use equivalence. Adding and subtracting fractions with denominators that are multiples of the same number (3b) uses a strategy whereby students change one of the denominators to match the other. Adding and subtracting two fractions with unlike denominators (3c) requires changing both denominators. These increasingly complex operations should thus be learned sequentially.

We are warned, however, of the implications of focusing only on high-level, externally-driven (and narrowly-interpreted - see section 2.2.2) curricula. Isoda (1996:106) reminds us that "because curriculum and students' development are mutually related, students' development reflects the curriculum and investigations of development cannot prove its hierarchy". Therefore WP1 is less concerned about the impact of the curriculum and more focused on children's thinking in the moment that influences their mathematical understanding and how this influences sequencing and switching. It is to this aspect the discussion now turns.

2.2 Five dimensions of fractions learning, a micro-level approach

In order to address the micro-level detail within the students' learning trajectory we have identified the following five dimensions of fractions learning that all tasks - exploratory and structured – reflect (see Table 1). This work took place in Y2 of the project as we became exposed to more literature, designed further tasks, and trialed them with students and teachers. We also undertook extensive iterations with key members of the consortium, particularly those involved in task-dependent and task-independent support in WP2 to ensure that the system:

- selects appropriate intervention strategies by understanding the reason for the erroneous student behaviour in tasks;
- assesses progress during tasks; and
- selects adequate interventions if learners get stuck.

It also influenced WP3 that has built a GUI framework that allows a family of engaging exploratory activities to be developed and eventually be encoded in the system (WP4). This work also provides significant input for sequencing and switching (D1.3). How the five dimensions introduced here are used in sequencing and switching is briefly discussed in Section 2.3 and in more detail in D1.3.

Table 2: Five dimensions of fractions learning

1. Fine-grain goals
2. Fraction interpretations
3. Fraction representations
4. Fraction types
5. Task types

2.2.1 Fine-grain goals

In a traditional ideal classroom a teacher would compare each student's learning outcomes to a predicted coarse-grain learning trajectory. (S)he would then identify appropriate tasks according to their content, whether they actively engage students in mathematical thinking, how they take into account students' previous knowledge and experiences, what tools should be used to support students' understanding of the mathematical concepts and what materials need to be provided to scaffold thinking about mathematical ideas (Anthony & Walshaw, 2007). In order to emulate this highly-complex decision-making process within the iTalk2Learn platform we had to go beyond just mapping how the exploratory and structured tasks link to the coarse-grain goals. In a similar way to what a teacher might do implicitly, we undertook the process of identifying the specific learning objectives for every exploratory and structured task in order to provide for the system the fine-grain goals (FGGs) contained within all the exploratory and structured tasks. These fine-grain goals are what the rule-based and machine-learning prototypes of sequencing and task-dependent support are based on.

We developed the fine-grain goals with a mind to Bloom's taxonomy of educational objectives (Bloom et al., 1956) and subsequent amendments to that original (Anderson et al., 1999; Marzano & Kendall, 2007). Drawing on Anderson et al's (2001) work, Mayer (2002) explains that meaningful learning occurs when knowledge and cognitive processes come together so we framed the FGGs in terms of two dimensions: a) subject content and b) a description of what is to be done with that content (Krathwohl, 2002; Melis et al, 2008). Following Anderson et al's (2001) advice that such goals are typically composed of a verb describing the intended cognitive process and one or more nouns referring to the knowledge that the students are supposed to acquire, we identified 14 FGGs that that exist in the exploratory and structured tasks. These can be seen in Table 3.

Table 3: Fine-grain goals for exploratory and structured tasks

1. Recognise the whole
2. Interpret the size of a fractional part
3. Attribute fraction representation to symbol
4. Recognise different representations that are the same but look different
5. Compare two fractions
6. Identify the factors of the numerator/denominator
7. Find the greatest common factor
8. Expand fractions to find equivalents
9. Multiply numerator and denominator to find equivalents
10. Generate a common denominator
11. Partition to find equivalents
12. Cancel down to find equivalents
13. Identify the relationship between the size of the piece and the number of pieces
14. Produce the sum of two fractions
15. Produce the solution of subtracting two fractions

2.2.2 Fraction interpretations

Seminal work in fractions by Kieran (1976, 1981) identified five interpretations of fractions: part-whole, ratio, operator, measure, and quotient. Researchers refer to these interpretations using five broad representations: symbols, area, number line, set of objects and liquid measures (Charalambous & Pitta-Pantazi, 2007; Kieren, 1976; Lamon, 2012; Pantziara & Philippou, 2012; Silver, 1983). Students tend to receive a limited number of interpretations in their curriculum diets with part-whole, the most common interpretation (Baturo, 2004). It is the extensive focus on the part-whole interpretation in curricula around the world that has led many researchers to focus on other interpretations and to question the extent to which part-whole interpretations impact on children's understanding of fractions (Behr, Lesh, Post, & Silver, 1983; Charalambous, Delaney, Hsu, & Mesa, 2010 ; Panaoura, Gagatsis, Deliyianni, & Elia, 2009). The development of each fraction subconstruct in isolation does not necessarily lead to understanding of the other interpretations (Brousseau et al., 2004, Charalambous & Pitta-Pantazi, 2007) and indeed, "to ignore those other ideas in instruction leaves a child with a deficient understanding of the part—whole fractions themselves, and an impoverished foundation for the rational number system, the real numbers, the complex numbers, and all of the higher mathematical and scientific ideas that rely on these number systems" (Lamon, 2012:33).

Each interpretation is so complex that often researchers study them in isolation. However, we are not studying each interpretation or representation at such a fine-grain level. Instead, we are extracting from the existing literature what students need to know and the difficulties associated with different interpretations and representations and we are applying this knowledge to develop a coherent conceptual framework (referred to as the fractions interpretations/representations matrix - See Appendix I) that underpins the iTalk2Learn learning platform and its associated tasks. As we designed each exploratory task and selected each structured task we identified the fraction interpretations that are used. This provides a broader diet to learners and supports a coherent system of learning as sequencing and switching ensures a wide range of interpretations are encouraged.

2.2.3 Multiple representations

In addition to fraction interpretations, there are also a number of ways that fractions can be represented. Graphical representations of fractions, such as area models (e.g., fraction circles, Geoboards) and linear models (e.g., fraction strips, Cuisenaire rods, number lines) are used extensively in fractions instruction. Several studies demonstrate the promise of providing instruction that links these representations of fractions to the underlying fractions interpretations (Kong, 2008; Paik, 2005; Pitta-Pantazi, Gray, & Christou, 2004; Yang & Reys, 2001) and support in actively making connections among the representations (Ainsworth, 1999; Tabachneck, Leonardo, & Simon, 1994). Indeed, the use of multiple representations in task design is well-documented. For example, the Task Type and Mathematics Learning (TTML) project identifies how tasks can be used to provide an "introduction to, or use of models, representations, tools, or explanations that elaborate or exemplify the mathematics" (Clark & Sanders, 2009). In these types of tasks there is no compulsion for teacher exposition because the models and representations themselves enable students to generate the mathematical ideas and justification. This is attractive to us because of the implicit goal behind systems like iTalk2Learn that need to operate effectively without significant teacher or adult input. We use multiple representations as objects for the students to act upon in order to construct mathematical meaning based on situated abstractions. By manipulating the representations, students appear able to generate their own strategies and reflect upon their own errors. Streefland (2012, pg. 3) identifies "flexible application of (visual) models and schemes in connection with clever calculations" as one of five indicators for describing students' increased mathematical knowledge and understanding.

To have a complete understanding of fractions requires an understanding of the different representations and how they interrelate (Kieren, 1979), but we must not overlook the close relationship between fraction representations and interpretations that exists. In Table 4 (overleaf) we demonstrate how the different fraction representations can change the way fraction interpretations are perceived. There is a significant lack of research literature related to some representations, particularly liquid measures, and so the table is by no means complete. The gaps in the literature around liquid measures are of interest to the project because there appears to be potential about their educational use (Silver, 1983). We already have some promising emerging data from our UK studies related to how students and teachers use liquid measures to learn and teach fractions and we intend to contribute to the fraction education research literature in this area. This is discussed in D5.2.

Table 4: Illustration of how the representations can provide a different context for fraction interpretations

Interpretation	*Representations*
Fractions as Part-Whole	In part-whole situations using the area representation, students understand that the denominator is the number of equal parts, the whole has been cut into and the numerator is the number of parts taken (Mamede, Nunes & Bryant, 2005). Where sets of objects are used, students understand that a set is partitioned into parts of equal size and that the numerator must be less than or equal to the denominator (Charalambous and Pitta-Pantazi 2007, citing Lamon, 1999 and Marshall, 1993).
Fractions as Ratio	In ratio situations, students understand that ratio is a comparison between two quantities and is therefore considered a comparative index (Carraher, 1996, cited in Charalambous and Pitta-Pantazi 2007).
Fractions as Operator	In operator situations involving sets of objects, students understand the relative nature of fractions. They realise that the same fraction symbol may actually refer to different quantities (e.g. ½ of 6 is not equivalent to ½ of 14) and that different fraction symbols may be equivalent because they refer to the same quantity (e.g. 1/2 and 2/4) (Nunes, 2006; Mamede, Nunes & Bryant, 2005). The relative nature of fractions involves multiplicative thinking, which is hierarchical in structure. For example, 4x3 involves thinking one 3s, [then] two 3s, [then] three 3s, [then] four 3s, but conservation also requires students to know that 1/4 = 3/12 at same time (Kamii & Clark, 1995). In operator situations involving the area representation, students understand that fractions are transformers (Lamon, 1999). They can take a figure in the geometric plane and map it onto a larger or smaller figure of the same shape (Charalambous and Pitta-Pantazi 2007). They are also referred to as a *stretcher/shrinker* (Behr et al., 1983). When using sets of objects, students understand that fractions can increase or decrease the number of objects in a set (Lamon, 1999). Students also understand that fractions can lengthen or shorten line segments (Lamon, 1999). This is related to the use of line representations.
Fractions as Quotient	In quotient situations, students understand a fraction symbol as the result of a division (Newstead & Murray, 1998). When using a set of objects, students understand that the denominator is the number of recipients and the numerator is the number of items being shared (Mamede, Nunes & Bryant, 2005).
Fractions as Measurement	Kilpatrick et al. (2001:235, cited in Clarke & Roche, 2009) commented that the simplest interpretation and use of fractions is within measurement, and it is so fundamental that it can be easily overlooked. In the context of measure, students seem to understand that fractions can be placed on a number line because they are numbers in their own right (Baturo, 2004).

2.2.4 Fraction type

The dimension of fraction type emerged from the formative evaluation studies with students and teachers (reported in D5.2). Unsurprisingly, we found that different students responded to the same exploratory or structured task differently. One significant difference was the extent to which each student found the tasks challenging, based on their prior knowledge and experience of fractions. In a project like iTalk2Learn, students bring a variety of unique previous experiences to bear in their learning with the platform. One of the ways we can address the finer-grain details of their knowledge is to offer different types of fractions (see Table 5) dependent on their level of attainment (i.e. confidence and competence in using the different types of fractions).

Table 5: Fraction types

Set A: Unit fractions	A unit fraction is a fraction where the numerator is one and the denominator is a positive integer. For example, 1/1, 1/2, 1/3, 1/8, 1/10.
Set B: Proper fractions	A proper fraction is a fraction where the numerator is less than the denominator. (Note: proper fractions include the set of unit fractions but for the purposes of our tasks we make the distinction). For example, 1/4, 3/4, 7/8, 19/20.
Set C: Improper fractions	A fraction in which the numerator is greater than the denominator. For example, 5/4, 6/2, 12/10, 6/5.

Typically, tasks involving Set A fractions will be less challenging than tasks with Set B fractions. Similarly, Set C tasks are likely to be more challenging than tasks using Set A or Set B fractions. Fraction types are made use of during sequencing and switching. This is introduced in Section 2.3 and discussed further in D1.3.

2.2.5 Task type

We were once again mindful of Bloom's taxonomy of educational objectives (Bloom et al., 1956) and later variations (Anderson et al., 1999; Marzano & Kendall, 2007) when we considered the types of task we would offer students in the platform. However, we found these taxonomies too general for our needs. We therefore created our own task type classification for mathematics education that meets the purposes of the project.

Regardless of student age and mathematical content, mathematics education research identifies very similar requirements for designing exploratory and structured tasks. Drawing from existing literature in elementary, secondary and tertiary mathematics age phases (e.g. Pointon & Sangwin, 2004; Sangwin, 2003; Stein et al, 2000; Swan, 2008, 2011) we have developed a new task classification (Table 6) that brings together the core elements identified by these researchers in order to specifically address our specific mathematics context within the project.

Table 6: Task classification

Tasks to encourage procedural learning	
Structured	Students execute a procedure or algorithm by reproducing rules, recalling memorised facts. There are no conceptual connections explicitly made. Tasks do not require detailed explanations to be made.
Tasks to encourage conceptual learning	
Classify	Students explore what is the same/different about a number of objects to begin to identify properties. They may produce further examples. Students formulate a basic definition.
Analyse/ Reason	Students analyse existing work, normally from another 'student' (real or fictitious) with a view to find errors and challenge students' own reasoning.
Interpret	Students will use multiple representations to model the context. They will use different forms of equivalent information in order to model the context.
Justify	Students will exemplify or refute a statement. Students will provide a justification for their argument. Primary students will work towards providing a general argument that requires abstract or general objects.
Construct/ Create	Students create problems or examples/instances. Students are encouraged to select their own approach to follow through; many approaches are possible.

By using the classification to support the design of the exploratory tasks for iTalk2Learn we are ensuring an appropriate range of experiences for students using the exploratory learning environment within the platform.

3 Misconceptions and errors

In section 2 we addressed the two parallel aspects we took to ensure a coherent system of fractions. These create two sides of a coin: the coarse-grain goals guide the student through a learning trajectory at the macro-level and the five dimensions of fractions learning do so at the micro-level. In this section we highlight another component of fractions learning which WP1 is providing as we develop the iTalk2Learn platform: the expert knowledge that informs the student model in WP2, including misconceptions, to adapt feedback to the student (see D2.2.1).

The iTalk2Learn project is interested in the misconceptions and errors that students make as they work within the platform. Unchecked, misconceptions and procedural errors can be a barrier to students' robust mathematical knowledge. However, focusing on misconceptions and errors has many benefits. These include developing successful learners who attempt more challenging work by adopting a "constructive attitude" to their mistakes (Koshy, 2000:173), develop a coherent mathematical knowledge (Barmby et al, 2009), and explore and discuss their misconceptions (Spooner, 2002).

In the platform we will use misconceptions (inferred by observing systematic errors students make in exploratory tasks) and procedural errors (mistakes that students make when working through structured tasks) in two ways. Misconceptions and errors can be used to develop students' conceptual and procedural knowledge by: a) planning systematic experiences (switching and sequencing – D1.3) to address them, and b) informing task-dependent feedback (D2.2.1).

Procedural errors are made as a result of a procedure being undertaken incorrectly. If a procedural error is made by a student but they have previously demonstrated conceptual understanding, they may require further practice of a particular aspect of the procedure. For example, practicing addition sums. These errors need to be addressed also, but they require a more procedural approach to remedying them. As such we had to take into account misconceptions and errors in the design of our tasks and this also has an impact on D1.3.

3.1.1 Fraction misconceptions

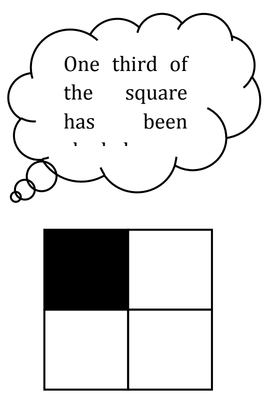
During Year 2 we have built up a significant database of common misconceptions related to fractions that we will use in the project. Some of these are from research literature and others arose during trials of Fractions Lab and the exploratory tasks. We have published these in the 3rd edition of a popular book used by trainee teachers and teachers across the UK (Hansen, 2014b). Since publication, our thinking about fractions misconceptions has moved on and we have now identified two classes of fractions misconceptions which we believe could be generalizable for all aspects of mathematics. There are some which we call 'global fraction misconceptions' that can be seen in numerous situations (such as treating the numerator and denominator as whole numbers rather than parts of a fraction). These are easily observed but are difficult to address because of their endemic nature. We call the other misconceptions 'situated fraction misconceptions'. Situated misconceptions might be driven by task content, task context, the representations being used or any combination of these. Therefore, in the appendix we identify for each situated misconception the interpretation and representation it relates to, as well as coarse-grain goal it is most likely to be seen within.

The notion of global and specific misconceptions is an original contribution to the mathematics education literature. Examples of global and situated misconceptions are provided below.

Table 7: Example of Global Misconception

<table>
<tr><th>Interpretation/ representation</th><th>Coarse-grain goal</th><th>Misconception</th><th>Commentary</th></tr>
<tr><td><table>
<tr><td></td><td>S</td><td>A</td><td>N</td><td>O</td><td>L</td></tr>
<tr><td>PW</td><td>■</td><td>■</td><td>■</td><td>■</td><td>■</td></tr>
<tr><td>R</td><td>■</td><td>■</td><td>■</td><td>■</td><td>■</td></tr>
<tr><td>Op</td><td>■</td><td>■</td><td>■</td><td>■</td><td>■</td></tr>
<tr><td>Q</td><td>■</td><td>■</td><td>■</td><td>■</td><td>■</td></tr>
<tr><td>M</td><td>■</td><td>■</td><td>■</td><td>■</td><td>■</td></tr>
</table></td><td><table>
<tr><td>1</td><td>■</td></tr>
<tr><td>2</td><td>■</td></tr>
<tr><td>3a</td><td>■</td></tr>
<tr><td>3b</td><td>■</td></tr>
<tr><td>3c</td><td>■</td></tr>
</table></td><td>Treating the numerator and denominator as if they were whole numbers</td><td>Students, relying on their whole-number constructs, do not understand that the numerator and denominator are not whole numbers but instead have roles within the number the fraction represents.
This is a significant global misconception and underpins many of the situated misconceptions listed below.</td></tr>
</table>

Table 8: Example of Situated Misconception

<table>
<tr><th>Interpretation/ representation</th><th>Coarse-grain goal</th><th>Misconception</th><th>Commentary</th></tr>
<tr><td><table>
<tr><td></td><td>S</td><td>A</td><td>N</td><td>O</td><td>L</td></tr>
<tr><td>PW</td><td>■</td><td>■</td><td></td><td></td><td></td></tr>
<tr><td>R</td><td>■</td><td></td><td></td><td></td><td></td></tr>
<tr><td>Op</td><td>■</td><td></td><td></td><td></td><td></td></tr>
<tr><td>Q</td><td>■</td><td></td><td></td><td></td><td></td></tr>
<tr><td>M</td><td>■</td><td></td><td></td><td></td><td></td></tr>
</table></td><td><table>
<tr><td>1</td><td>■</td></tr>
<tr><td>2</td><td></td></tr>
<tr><td>3a</td><td></td></tr>
<tr><td>3b</td><td></td></tr>
<tr><td>3c</td><td></td></tr>
</table></td><td>
</td><td>There are several reasons why students make this error. The most likely reason is that that they don't see the four parts as all part of the whole. Understanding the whole and that fractions are a part of the whole is very important, but here, students may focus on the individual parts of the square.</td></tr>
</table>

3.1.2 Errors and misconceptions usage in the project

While we are mindful of global misconceptions we are aware that addressing them involves cognitive change that takes a significant length of time. For example, the most common and most significant barrier to fractions learning is treating numerators and denominators as whole numbers (see Table 11). This misconception inevitably evolves for students over a period of years as they learn to reason with whole numbers without due attention to rational number constructs. We cannot begin to address these global misconceptions in the project. However, we can address situated misconceptions. The example in Table 12 is a situated misconception that is underpinned by the global misconception discussed here, but it is possible to challenge a student's thinking about how they perceive the representation and record it using symbols building a situated abstraction. We do this by the very careful and challenging process of exploratory task design, ensuring every task "give[s] rise to contradictions or surprises. In these tasks, learners need to sort out what is happening, resolve differences of opinion or conflicting explanations, and find some way to account for what is going on. Learners are called upon to explain things to each other and to locate differences and agreements in their explanations" (Mason and Johnson-Wilder, 2006:64). A detailed discussion of our exploratory task design can be found Section 4.

By being aware of the situated misconceptions that are more likely to be exhibited when undertaking particular tasks, it is possible for the system to provide feedback through the task-dependent support (WP2). This information is also used to provide more domain information to the switching and sequencing engine. Within D1.3 there is further discussion about how conceptual misconceptions are used and addressed in switching and sequencing.

4 Task design for robust elementary mathematics knowledge

Within iTalk2Learn we are interested in how exploratory tasks encourage conceptual learning and how these can be interwoven with structured tasks to provide an intelligent tutoring system that promotes robust mathematical knowledge (see D1.1 and D1.3 for further discussion). As a result, we have taken a very detailed approach to structured task selection and exploratory task design to ensure the best possible outcome. In addition to ensuring the exploratory tasks meet the requirements for a coherent system of fractions knowledge (the CGGs and the five dimensions of fractions learning), we have carefully selected structured tasks that also meet these standards. This is discussed in Section 4.3.

4.1 Procedural learning: using structured tasks from existing content from Maths-Whizz and Fractions Tutor

As discussed in D1.1, we are utilising existing structured mathematics tasks from Maths-Whizz (UK) and Fractions Tutor (Germany). We expect the students to undertake procedural learning using these tasks. In order for the iTalk2learn platform to select (recommend) subsequent tasks for each student based on their interaction, a variety of tasks is required.

4.1.1 Maths-Whizz

Maths-Whizz content is delivered in three stages: a Teaching Page, interactive exercises and a short test. When introduced to a learning objective, students first see a short introduction (the Teaching Page) which explains, procedurally, how to complete the exercise successfully. They then move onto questions in the interactive exercise, which provide students with guided instruction and immediate feedback from structured tasks. As students work through the questions, they receive feedback according to their answers. When an incorrect answer is entered, Maths-Whizz provides feedback in the form of a *help*, encouraging students to elaborate and reflect about problem-solving strategies before having another attempt. Up to three helps are offered per question, at which point a student receives the correct answer (see Fig. 1 for an example). Correct answers are rewarded with a celebratory response. Following an exercise, students are required to demonstrate their understanding in short tests, where no helps are available. The Maths-Whizz sequencer guides students through the curriculum, selecting exercises and tests across multiple topics and learning objectives based on students' prior progress (see D1.1).

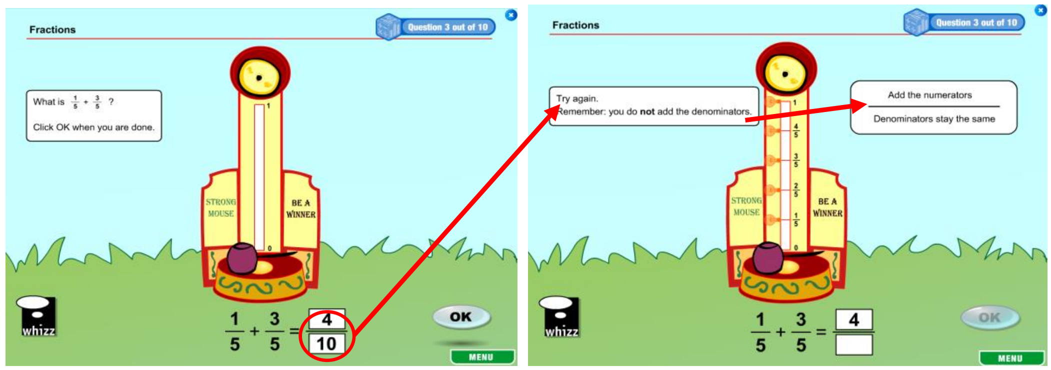


Figure 1. A Maths-Whizz question. A student's response where like denominators have been incorrectly added together results in feedback that states, "Remember: you do not add the denominators" and is followed up with "Add the numerators; Denominators stay the same".

Maths-Whizz exercises use a range of graphical representations such as circles, rectangles, number lines, liquid measures and symbols within contexts that the students may be familiar. For English students, tasks are aligned to the Mathematics National Curriculum of England and associated guidance (such as The National Numeracy Strategy and the National Primary Framework) that schools follow. The tasks and their interplay with the exploratory tasks is presented in Appendix III in detail.

4.1.2 Fractions Tutor

As described in D 1.1 the Fractions Tutor (FT) is a web-based Cognitive Tutor for learning fractions (Rau, Aleven, & Rummel, 2009; Rau, et al., 2013; Rau, Aleven, Rummel, & Rohrbach, 2012; Olsen, Belenky, Aleven & Rummel, in press). It covers a range of 10 different topics (i.e. units). Given that one of the strong features of FT is its well-researched and developed approach to teaching students a procedural knowledge of equivalent fractions, we particularly (but not exclusively) focus on this aspect in Germany. As it is generally the case in FT the students will solve problems step-by-step and receive immediate feedback (see Figure 2).

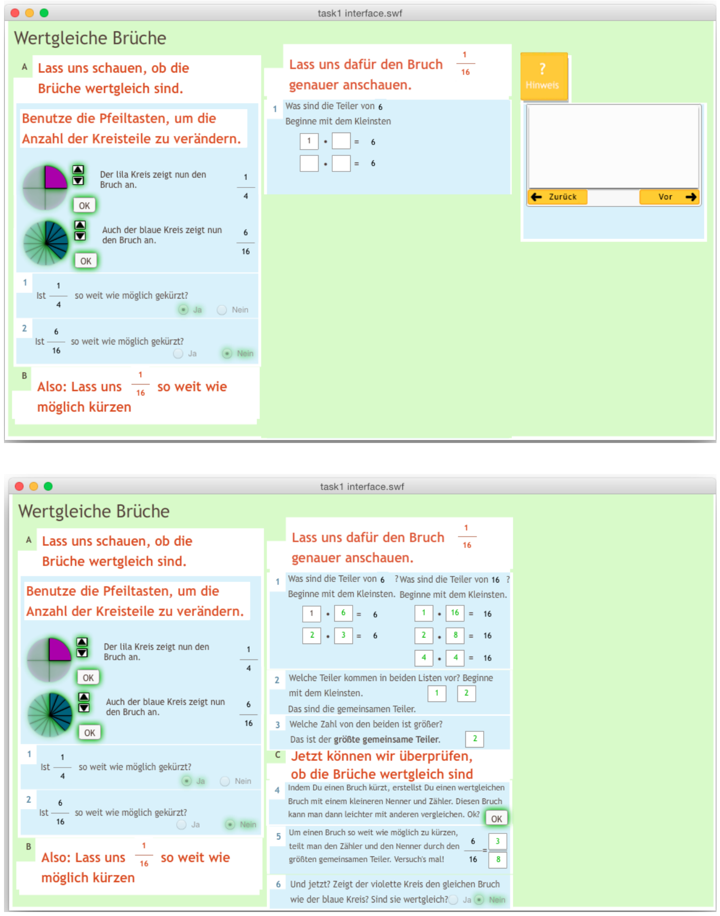
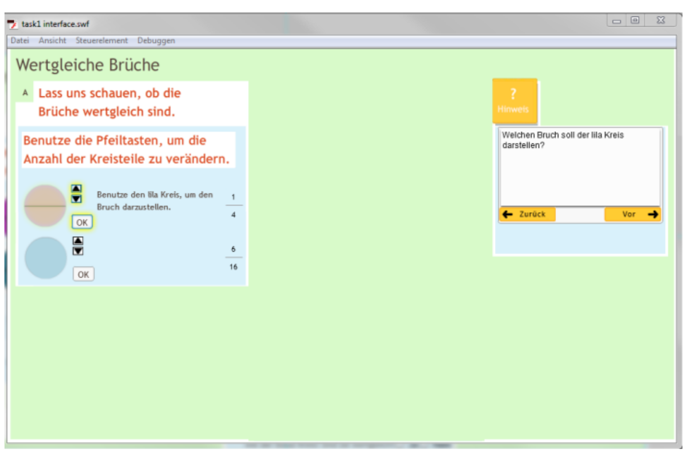


Figure 2: Step-by-step feedback in Fractions Tutor

Additionally, FT help functionalities allow students to ask for hints on up to three different levels: abstract, concrete and solution (Figure 3).



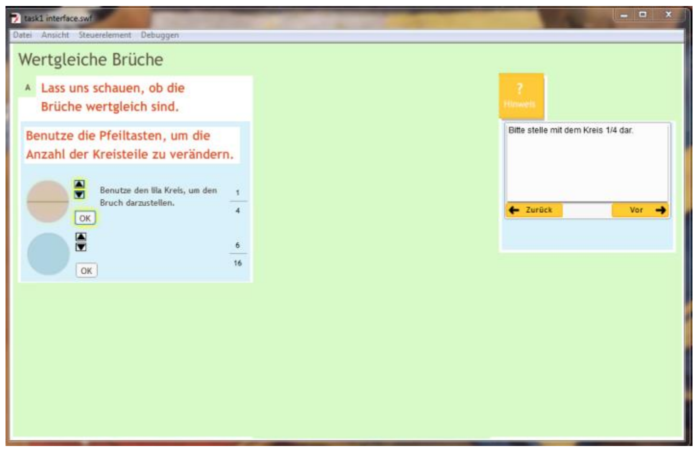


Figure 3: Three levels of hints in Fractions Tutor (abstract, concrete, solution)

In Appendix III we provide a list of FT tasks and how they relate to the CGGs and the five dimensions of fractions learning.

4.2 *Conceptual learning: the design of exploratory tasks*

In this section we provide details of how the exploratory tasks for conceptual learning to encourage conceptual understanding were developed over the first 24 months of the project. Henningsen & Stein (1997) explain that designing tasks that enable high-level thinking is not without difficulty. First, tasks convey the messages about how we want students to think and act, but this may introduce a type of socio-mathematical norm (Yackel & Cobb, 1996) that the student has not met before. Furthermore, high-level tasks are complex and take more time to complete than routine activities (Pointon & Sangwin, 2004) and so there is more likely to be a decline in students' engagement to the more demanding thought processes required.

Using the same design principles as those for the ELE, we used three elements to design the tasks: design conjectures arose from experience with related tasks, design drivers arose from literature, and design assumptions arose from the designers' pedagogical approaches. This is presented schematically in Figure 3 and described in relation to the coherent system for fractions learning (CGGs and five dimensions) in Table 13.

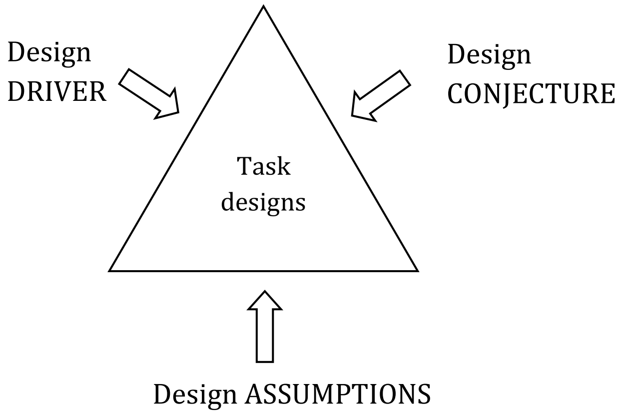


Figure 4: The elements of task design (based on Hansen, 2008)

4.2.1 The elements for designing exploratory tasks and how they are utilised in iTalk2Learn

We began the process of task design using the **design drivers** to act as principles that guided the design of the tasks. To identify the task design drivers we undertook a literature search. This identified for us the Coarse-Grain Goals (CGGs; section 2.1), the interpretations to use (section 2.2.1), the representations to include (section 2.2.3), the task types to design (section 2.2.5). In addition to this a detailed trawl through the research literature, mathematics education books and text books we identified the mathematical terminology and associated phrases / utterances that students may use in relation to fractions. This list of 279 English terms and its duplicated set of German terms with in excess of 1650 accompanying utterances for English plus their parallel German utterances directly informed WP3's work on speech recognition (in particular the development of the language model) and will also be used for evaluations regarding precision and recall (the results will be published in D3.3.2). They also inform WP2's work, in particular the task-dependent feedback.

Design conjectures are generic and specific conjectures about the design of tasks and their effectiveness, which arise from the critical analysis of previous experience with related educational tasks or from evidence from trials with previous tasks or early prototypes during the design process. We trialed early exploratory tasks with students from the target age range. This particularly informed the decision to include fraction types (section 2.2.4). We also sought feedback about the user interface and interactivity (D3.2) and used students' comments during that time to consider how the representations should behave and could be manipulated in tasks.

Design assumptions arise from personal knowledge and understanding. We drew on our own "professional artistry" (Schön, 1983) to identify how tasks might be completed by students and how the system would know that a task was complete. This was challenging within an exploratory learning environment where there is no traditional mechanism for letting the system know the task is finished. We also worked with domain experts (see D7.3.1) to gain feedback about the tasks and to consider how the tasks could address the CGGs and misconceptions.

4.2.2 Exploratory task design

The exploratory tasks were designed by taking account of the affordances and constraints of Fractions Lab, the ELE, in addition to the three elements above (see D3.2 for further discussion of relationship between D1.2 and this WP). Throughout all formative evaluation studies we tested and revised the tasks (see D5.2) in an iterative fashion. As a result we developed a highly-honed set of exploratory tasks using a robust set of components that: address the CGGs; have clear tags related to the five dimensions of fractions learning; clearly identify the possible situated misconception(s) that students may exhibit and the global misconception(s) that underpin them; how students may complete the task; the possible opportunities and difficulties that may be presented to students while completing the task and suggested feedback in relation to this; and the mathematics terminology a student would be expected to use. The template that was used to design the tasks is explained in the next section.

An overview of all tasks and a representative task from each Coarse-Grain Goal is in Appendix IV.

4.3 Bringing exploratory and structured tasks together for robust mathematical learning: Preparation for sequencing and switching

We discussed in detail in D1.1 about how procedural and conceptual tasks develop students' robust mathematical knowledge and we extend this in D1.3 to provide the cognitive model for how switching and sequencing occurs. We present here the work we have undertaken to inform D1.3 and to enable switching and sequencing in practice (WP4) (see D4.2.1).

4.3.1 The task design template

In Section 4.2.2 we introduced the complex set of components we utilised as we designed the exploratory tasks. It demonstrates how designing high quality tasks is not a trivial process. In parallel, we undertook the process of selecting the structured tasks for use in the platform and began by categorising each structured task according to how it related to the CGGs (this was published in the appendices of the M18 version of this deliverable). However, as we progressed our discussions related to the cognitive model for switching and sequencing it became apparent that following this macro-level approach was not detailed enough (see sections 2.1 and 2.2 for further discussion) and we identified how the five dimensions for fractions learning are also addressed by the structured tasks (see Section 2.3). At the end of Year 2 we have worked iteratively with colleagues to the extent to which we can now see the value in using the same components to analyse the structured tasks as we have been able to with exploratory tasks. Appendix III provides an analysis of the structured tasks according to the coherent system of fractions learning (CGGs and the five dimensions). Switching and sequencing (D1.3) and its corresponding implementation (WP2) will encode and utilize this information to enable task-dependent, task-independent support (including the use of speech to detect terminology) and the delivery of appropriate tasks.

An overview of the task template is provided in Figure 4. Populated examples of structured tasks for switching and sequencing will be provided in D1.3. Examples of populated exploratory tasks can be found in Appendix IV.

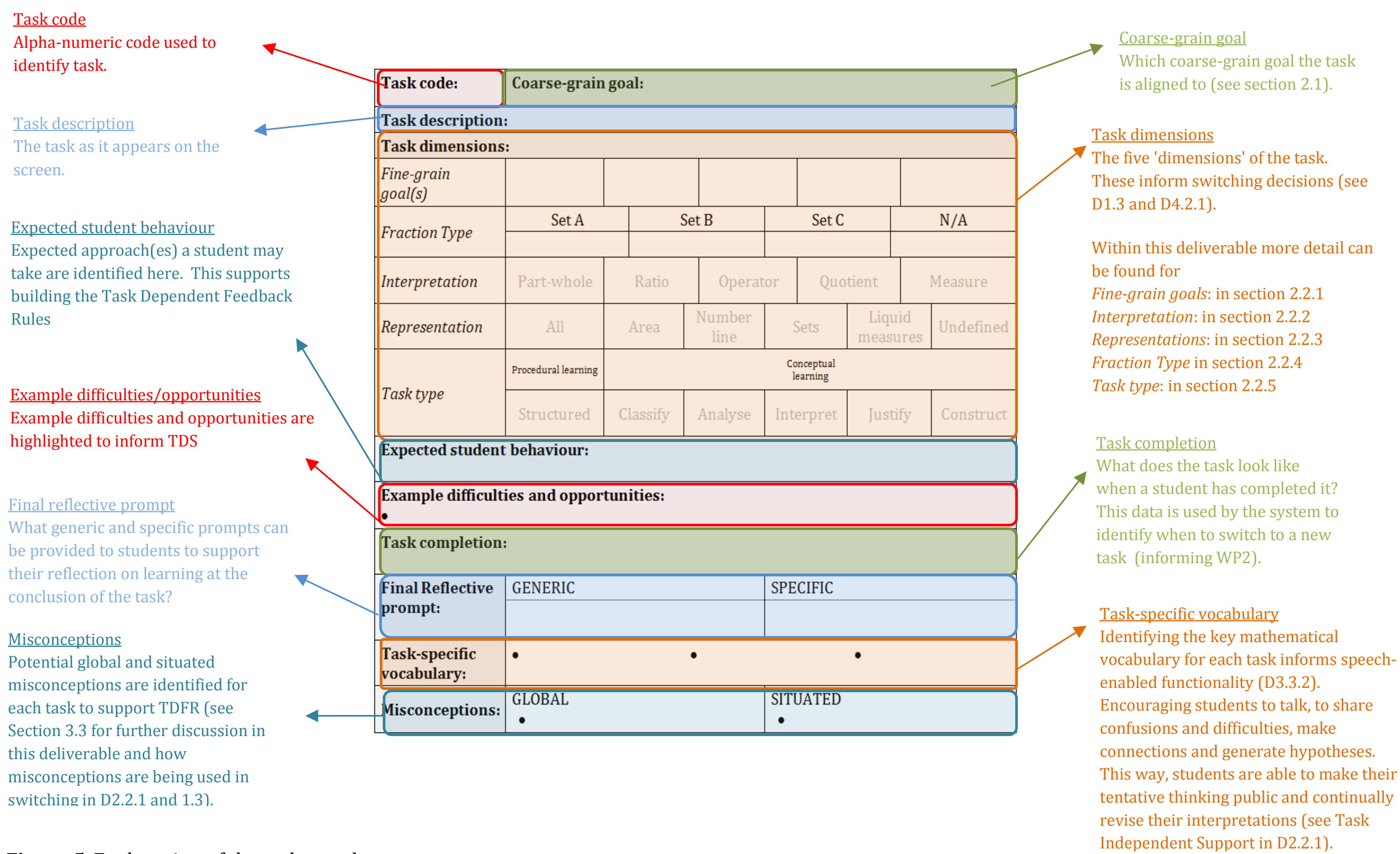


Figure 5. Explanation of the task template components.

5 Summary and next steps

This deliverable reports on the project tasks that develop appropriate content for the intelligent tutoring system and exploratory activities and align them appropriately (T1.2), and the identification and operationalization of relevant topics, learning objectives and problem solving strategies for elementary mathematics (T1.3).

We summarise below the three key contributions of this deliverable within and beyond the project. WP1, and this deliverable in particular had a 'service' role in the iTalk2learn project, that concludes in M24. However, we have been able to contribute to both practioner- and reseach-oriented venues, indicating the potential for this work. As such we outline next steps with respect to potential contributions.

5.1.1 Coherent system of fractions learning

The coarse-grain goals are based on curricula from around the world and as such do not, alone, offer anything new. However, they guide each student's learning trajectory over the length of their time using the iTalk2Learn project. The offer to the mathematics education literature comes with the combination of the coarse-grain goals with the five dimensions offering a coherent system of fractions learning that is a unique conceptualisation of fractions learning and teaching within the domain. Within the project this is underpinning/informing the work of WP2 and in particular the TDS . and the sequencing and switching engine that will be able to provide the appropriate task.

Furthermore, elaborating on the five dimensions could offer to the literature and pedagogy. For example, much has been written about interpretations and representations but the unique offer of iTalk2Learn to mathematics education is the matrix which brings these two components together. This is already well underway and has been welcomed by mathematics education and teachers alike. The task types amass existing literature from elementary, secondary and tertiary levels to provide one unified set of task types that can be used in the iTalk2Learn platform by teachers from any age phase. We intend to publish the task types classification by illustrating it using the tasks of the iTalk2Learn platform.

5.1.2 Misconceptions and errors

Our work in this area has been ground-breaking in two ways. First, we have assembled a large list of misconceptions related to fractions and have published these (a copy of the chapter is already available as a pdf file on the iTalk2Learn website). Second, we have identified two classifications of misconceptions: global misconceptions and situated misconceptions. We will explore these notions further through project evaluations in Y3. As detailed in D6.3.2 these evaluations have the potential to lead to high quality publications in educational research.

Within the project the exploratory tasks have been written with a view to exposing students' conceptual misunderstandings and our identification of misconceptions and errors informs WP2' student model for providing appropriate feedback. Sequencing and switching uses this information to provide appropriate content depending on students' needs (see D1.3).

5.1.3 Task design and selection

Finally, this WP has developed a task template that comprises a number of components related to structured and exploratory tasks. To our knowledge, the detailed database this template provides is unparalleled in other intelligent learning systems. Not only does it reflect the coherent system of learning by identifying how each task relates to the coarse-grain goals and five dimensions of fractions learning, it also encompasses a wider set of components related to pedagogy such as mathematical terminology and reflective prompts.

Whilst officially the task-design work concludes with this deliverable, task-related aspects will still concern the project as tasks are aligned with parallel work in WP2, WP3 and WP4 particularly with respect to any technical representation needs and metadata of the content.

The elements of task design (design drivers, conjectures and assumptions) have been used within the project to design the user interface (D3.2), the intervention model (D1.3) and the tasks reported in this deliverable. This principled approach to design and the parallels we draw with it in other projects has already contributed in our capacity-building activities (see D6.3.2) and also has the potential to contribute further in the field of design-based research

To conclude, not only is the work of this WP critical to the work of all other WPs, it also informs the mathematics education about high quality teaching and learning of fractions and the educational technology community about possible ways of operationalizing and describing exploratory tasks. Fractions are a difficult but crucially important aspect of mathematics to master and we offer, as an aside to the project, the benefits that our work brings to teachers. As WP5 reports in more detail, teachers have been involved explicitly in this WP (and T1.2 specifically) to influence task design and re-design (see D5.2 and D7.3.1). The teachers involved have endorsed our work and recognized the role the exploratory learning environment had on their own pedagogical subject knowledge. We also sought feedback from students and used the formative evaluations to establish the impact the ELE had on their conceptual understanding of fractions. This has helped us to become more explicit about the knowledge represented in the system, verifying the coherent system of fractions created, to be tested further in the Y3 formative and summative evaluations.

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Appendix I: The iTalk2Learn matrix

The matrix is used in a number of ways within the project. For example,

- the exploratory task design process has drawn upon the matrix to ensure a range of experiences for students;
- students' fractions misconceptions have been mapped to the matrix to identify context-specific misconceptions;
- task-dependent feedback is based on misconceptions related to representations;
- switching and sequencing is being informed by the matrix to ensure suitable task selection;

Not all of the matrix will be used in this project because the focus is on fraction addition and subtraction and some of this matrix relates to other fractions aspects such as multiplication and division. However, it is presented here its entirety for sake of completeness.

Table 9: The interpretations and representations matrix as published in Hansen, A. & Leeming, J. (2014) *Fractions, decimals, percentages, ratio and proportion.* In Witt, M. (ed) (2014) Primary mathematics for trainee teachers. London: Learning Matters/SAGE.

	Symbolic	Area/region	Number line	Sets of objects	Liquid measures
Part-whole	$\frac{3}{4}$	$\frac{3}{4}$ of the area is shaded	0 $\frac{3}{4}$ 1 $\frac{3}{4}$ of the number line is grey	$\frac{3}{4}$ of the objects are shaded	$\frac{3}{4}$ of the jug is filled
Ratio	$\frac{3}{4}$	3 out of 4 parts are shaded	0 $\frac{3}{4}$ 1 3 out of 4 parts have jumped along the number line(3 x $\frac{1}{4}$)	3 out of 4 objects are shaded	3 out of 4 parts of the liquid is water (the rest is oil)
Operator	$\frac{3}{4}$	Finding $\frac{3}{4}$ of the region gives:	0 1 Finding $\frac{3}{4}$ of the line segment gives: 0 $\frac{3}{4}$ 1	Finding $\frac{3}{4}$ of the objects gives:	Finding $\frac{3}{4}$ of the liquid gives:
Quotient	$\frac{3}{4}$	←3 are shared by 4→ 1 2 3 / 1 2 4 / 1 3 4 / 2 3 4 So each person receives $\frac{3}{4}$ each	e.g. Road relay String	3 objects are shared by 4, so each person receives $\frac{1}{4}$ of each object $= \frac{3}{4}$ each	3 jugs are shared by 4, so each new jug receives $\frac{1}{4}$ of each = $\frac{3}{4}$ jug
Measure	$\frac{3}{4}$	The shaded object is $\frac{3}{4}$ the white object	0 1 The second line segment is $\frac{3}{4}$ the first 0 $\frac{3}{4}$ 1	A B Set B is $\frac{3}{4}$ of Set A	The second jug is $\frac{3}{4}$ the first

Appendix II: Common misconceptions related to fractions

The misconceptions listed here are global misconceptions (GM) and situated misconceptions (SM) related to fractions.

In column two the matrix (see Appendix I) is used to show in which interpretation(s)/representation(s) each misconception may appear. In column three the coarse grain goal(s) that each misconception is likely to be observed within is highlighted (see Section 2.2 for discussion of coarse-grain goals). Note that by definition, global misconceptions have all interpretations, representations and coarse-grain goals selected.

Global misconceptions (GM)

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<tr><th>No.</th><th>Interpretation/representation</th><th>Coarse-grain goal</th><th>Misconception</th><th>Commentary</th></tr>
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<td>Using incorrect language to name fractions. E.g.
"One two-th" for one half
"Two tens" for two tenths
"Two and three" for two thirds
"Two slash three" for two thirds</td>
<td>Furani (2003) explores how naming and misnaming involve logic and rules, and are often an aid to supporting students' mathematical learning. Unfortunately, there are inconsistencies in the English conventions of naming fractions and this can be confusing. Indeed, in American English, one-quarter is referred to as 'one fourth'. Students need to learn that we use the term 'half' to represent 1 out of 2. A common procedural error with naming fractions is the use of 'one whole'. Sometimes students interpret this as 'one hole'.
Students may simply not have the denominator vocabulary (e.g. 'third') to</td>
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<td>read the fraction accurately. It is likely that they see the numbers in the fraction as two unrelated whole numbers separated by a line, rather than as a (fractional) number in its own right.</td>
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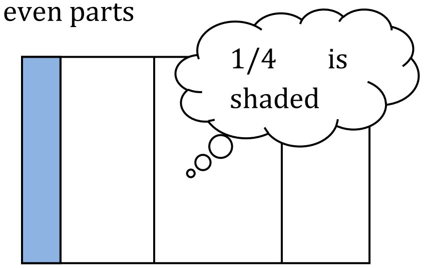
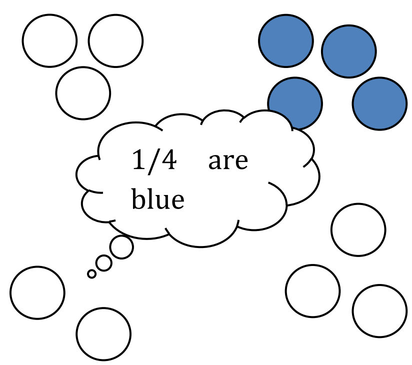
<td>Not dividing a region/set into even parts
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<td>In this misconception students do not understand that all the parts must be equal. They may not have had sufficient experience of dividing different representations into equal parts where the parts can be directly compared to each other. Students who have experienced limited representations may overgeneralise, for example dividing squares, oblongs and circles may work, so they believe the methods used for these shapes work for all shapes/representations.

Here, students do not understand that all the parts must be equal. They may not have had sufficient experiences of sharing equally. Students who have had limited experience with sets may exhibit this misconception.</td>
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		S A N O L PW R Op Q M	1 2 3a 3b 3c	Treating the numerator and denominator as if they were whole numbers	Students do not understand that the numerator and denominator have roles within the fraction symbol. This is a significant global misconception and underpins many of the situated misconceptions listed below.
		S A N O L PW R Op Q M	1 2 3a 3b 3c	Thinking a fraction is always less than 1. Thinking a fraction cannot be less than 0	Experiences that students had as young children and in primary school may lead to their thinking that a fraction is always part of one whole and therefore cannot be greater than one. This is a limitation of teaching using only the part-whole interpretation which reinforces this way of thinking.
		S A N O L PW R Op Q M	1 2 3a 3b 3c	Thinking fractions are only parts of shapes and not numbers in their own right	This misconception arises from students using fractions in their own experiences (e.g. cut the apple in half) and through school-based tasks involving part-whole representations.

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<tr><td colspan="5">Situated misconceptions (SM)</td></tr>
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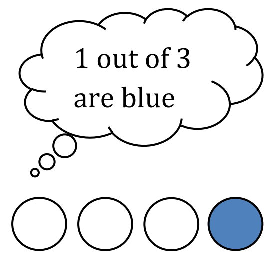
<td>Treating the part of the whole as a ratio

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<td>Students compares the one blue counter against the three white counters and concludes that one-third of the set of objects is blue. This misconception may be common in children up to about seven years of age and may be related to Piaget's findings from his class-inclusion task. Piaget found that if children are presented with a set of say five red objects and two blue objects and asked if there are more red objects or more 'objects', children will say there are more red objects. This is because they compare the red objects with the blue objects instead of comparing the red objects with the total number of objects in the set.</td>
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<td>Reading a fraction as 'x lots of' the fraction shown by the denominator and never as the denominator divided by x. E.g. Reading 3/4 as 3 lots of a quarter and never as a quarter of 3.</td>
<td>This is a situated misconception related to the global misconception of thinking about fractions only as shapes and not as numbers in their own right. It arises as a result of experience only with shapes.</td>
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<td>Placing 1/2 half away along the number line regardless of its spread instead of between 0 and 1</td>
<td>When students are introduced to fractions they are introduced to unit fractions such as one-half or one-quarter. They sometimes believe that a fraction is a number smaller than one, i.e. between 0 and 1. Students can usually successfully identify fractions on a number line between 0 and 1. The difficulty arises when the number line is extended to include numbers greater than 1. It may be that that the student's previous experience has involved halving shapes, and when a shape had been halved, there has been a line drawn in the middle. The student may apply this knowledge to the number line, instead of treating ½ as a number in its own right with a specific place on the number line half way between 0 and 1.</td>
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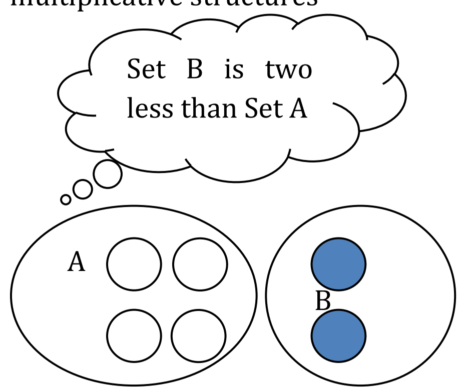
<td>Using additive instead of multiplicative structures

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<td>The correct response should be that set B has half as many dots as set A. Students draw on their knowledge of addition and subtraction from earlier in their education – when comparing the size of two sets of objects involved using addition/subtraction. Students may do this because they do not fully understand the nature of the task. This type of error may also arise when comparing lengths. For example, students may say that object A is</td>
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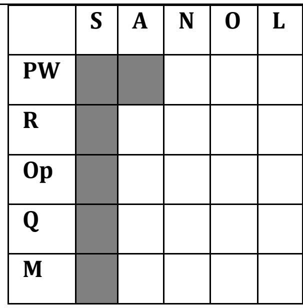
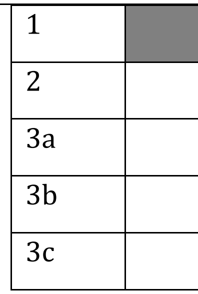
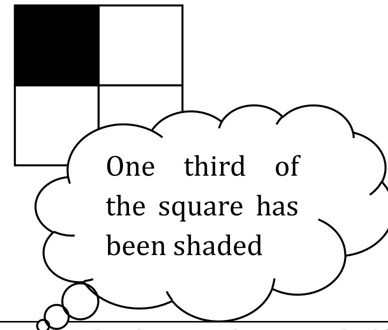
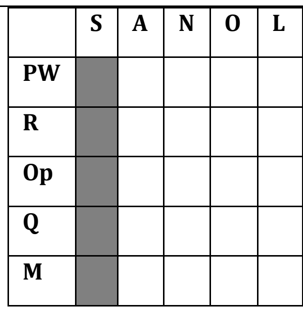
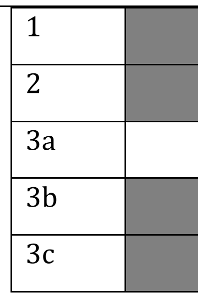
<tr><td></td><td>
</td><td>
</td><td>
</td><td>There are several reasons why students make this error. The most likely reason is that that they don't see the four parts as all part of the whole. Understanding the whole and that fractions are a part of the whole is very important, but here, students may focus on the individual parts of the square.</td></tr>
<tr><td></td><td>
</td><td>
</td><td>"One-third is bigger than one-half."</td><td>Students may use whole-number knowledge to order fractions and concludes that the denominator of one-third is larger than the denominator of one-half.

In this case, students over- generalise whole-number concepts and do not understand the written notation of fractions. They need to know what the notation means – that we have one part out of three equal parts.

Associated with this idea, students also need to realise that the vinculum (the fraction bar) in a fraction represents division. So means 3 out of 4, which is also 3 divided by 4.</td></tr>
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	S A N O L PW R Op Q M	1 2 3a 3b 3c	"3/6 is equivalent to 1/2. 8/4 is equivalent to 1/2."	Here the student has noticed the relationship between 3 and 6 and correctly noted that the fraction 3/6 is equivalent to 1/2. However, they have overgeneralised the relationship by assuming that the relationship can be applied the other way around too.
	S A N O L PW R Op Q M	1 2 3a 3b 3c	Changing only the denominator when making an equivalent fraction in order to add it to another fraction. E.g. In 2/3 + 1/6 the 2/3 is changed to 2/6 (instead of 4/6) to incorrectly add 2/6 + 1/6.	The student is aware that the denominators must be the same but they are treating the numerator and denominator as numbers that have not relationship between them and have not made an equivalent fraction. This misconception can occur for either fraction in the equation.
	S A N O L PW R Op Q M	1 2 3a 3b 3c	Student does not find the lowest common denominator when finding equivalent fractions to add or subtract two fractions. E.g. 1/3 + 2/5 = 10/30 + 12/30 = 22/30 instead of 1/3 + 2/5 = 5/15 + 6/15 = 11/15	This method provides a solution that would be satisfactory if 22/30 was cancelled down to the smallest equivalent fraction. However where another answer remains this is an inappropriate answer to provide.

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<td>The student explains that you find an equivalent fraction by multiplying the fraction. E.g. 9/12 x 3 = 27/36</td>
<td>This may simply be an inelegant explanation but it could be also that the student believes that the fraction is being made three times bigger by multiplying it by three.</td>
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<td>"One-quarter of a million is bigger than one-half of a thousand."</td>
<td>There the student has ignored the size of the whole and instead focused on the fraction itself, out of context.</td>
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<td>A: "3/5 is smaller than 5/16 because 5 is smaller than 16"

B: "4/5 is larger than 1/3 because 4 is bigger than 1"</td>
<td>For example A: It is likely that students focus only on the numerator and dismiss the denominator, to show that 3 is smaller than 5. For example B: Students focus only on the denominator to show that 4 is greater than 1. In both examples students show a lack of understanding of fractions, treating the two parts of each fraction as a whole number. The students need to know what the notation means – that we have x parts (the numerator) out of y equal parts (the denominator).

Associated with this idea, students also need to realise that ‘the line’ in a fraction</td>
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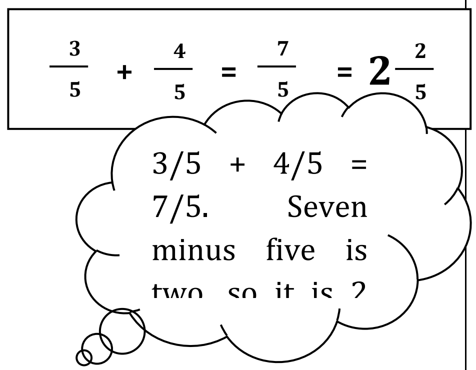
</td><td>
</td><td>Students use subtraction instead of division to work out the mixed number. This can happen when they think of the numerator and denominator as whole numbers, rather than parts of a fraction.</td></tr>
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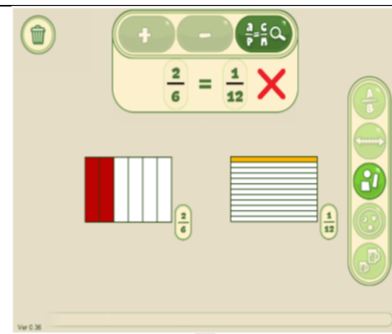
</td><td>

The student states the computer is wrong because 2 x 6 = 1 x 12.</td><td>Students treat the numerator and denominator as if they were whole numbers. They know that 2 multiplied by 6 is equal to 1 multiplied by 12, and is treating the vinculum (the fraction bar) as a multiplier.</td></tr>
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<td>"What you do to the top you do to the bottom"</td>
<td>Students can overgeneralise this multiplication procedure and apply it to other circumstances where it is inappropriate such as addition and subtraction.</td>
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<td>"3/4 = 1/12 because 3x4 = 1x12"</td>
<td>Students notice a coincidental relationship (in the example, that 3x4 = 12), and apply that coincidental relationship to make an answer.</td>
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<td>"6/7 = 8/9 because 7 take 6 is one and 9 take 8 is one".</td>
<td>Students treat the numerator and denominator as if they were whole numbers. They know that the difference between 6 and 7 is 1, and the difference between 8 and 9 is also 1. Therefore, they conclude that they are equivalent.</td>
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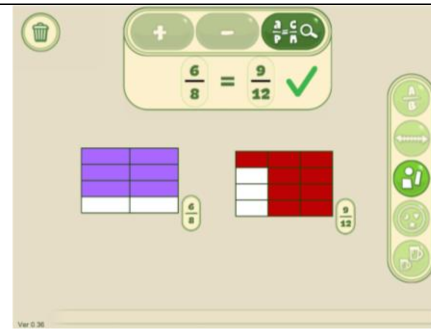
<td>
A student is surprised that 'FractionsLab' is right when it shows that 6/8 and 9/12 are equal.</td>
<td>Students treat the numerator and denominator as if they were whole numbers and using addition to try and explain the relationship between the two fractions. This is common, and students should be encouraged to think about using multiplicative structures to explain relationships between fractions rather than additive structures.</td>
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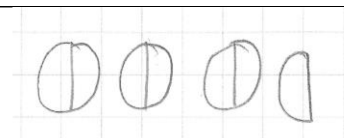
<td>
A student is asked to find the mixed number equivalent to 7/2. The student responds, “Seven halves are 3.1 because there are three wholes and one left over”</td>
<td>Students have not used 0.5 for 1/2. This may be because using remainders in whole number division is more familiar to students and they have applied that knowledge in this situation. Students may not know that ½ = 0.5 (or that 0.1 = 1/10) in this context, and places what they deem to be the most sensible solution: that 0.1 represents one left over.</td>
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1/2 + 1/4 = 1/6</td><td>The numerators are multiplied and the denominators are added by students.</td></tr>
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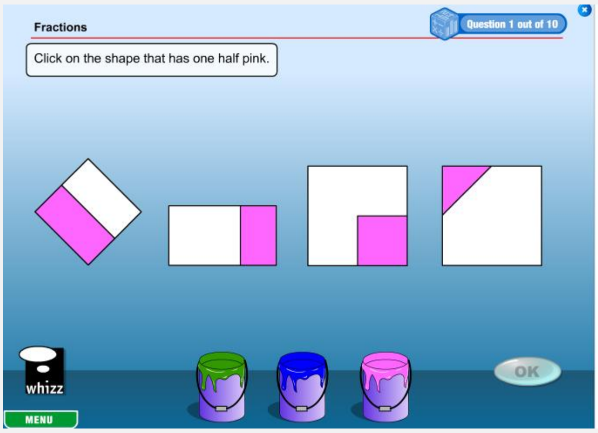
Appendix III: Structured Tasks in Whizz and Fraction Tutor

In this appendix we present the structured tasks analysed according to the task dimensions as discussed in Section 4.3 of the deliverable. In Part 1 each standalone Maths-Whizz task is analysed and in Part 2 the Fractions Tutor categories of tasks are analysed.

Part 1: Maths-Whizz tasks

Task code: MA_GBR_0700CAx0100	**Coarse-grain goal: Fractions as part of a whole**

Task description:



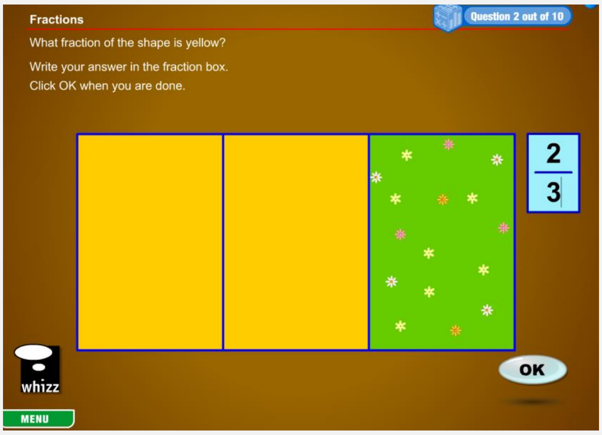
Task dimensions:

Fine-grain goal(s)	2. Interpret the size of a fractional part	4. Recognise different representations that are the same but look different			
Fraction Type	Set A	Set B	Set C	N/A	
	✓				
Interpretation	**Part-whole**	Ratio	Operator	Quotient	Measure

Representation	All	**Area**	Number line		Sets	Liquid measure	Undefined
Task type	Procedural learning	Conceptual learning					
	Structured	Classify	Analyse	Interpret		Justify	Construct

Task code: MA_GBR_0825CAx0100	**Coarse-grain goal: Fractions as part of a whole**

Task description:

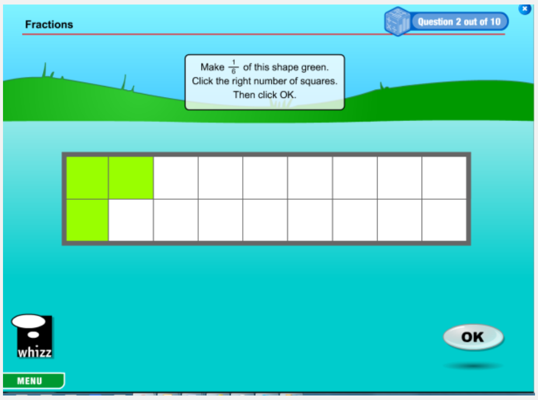


Task dimensions:

Fine-grain goal(s)	2. Interpret the size of a fractional part	3. Attribute fraction representation to symbol	4. Recognise different representations that are the same but look different	11. Identify the relationship between the size of the piece and the number of pieces		
Fraction Type	Set A	Set B	Set C	N/A		
		✓				
Interpretation	**Part-whole**	Ratio	Operator	Quotient	Measure	
Representation	All	**Area**	Number line	Sets	Liquid measure	Undefined
Task type	Procedural learning	Conceptual learning				
	Structured	Classify	Analyse	Interpret	Justify	Construct

Task code: MA_GBR_1200CAx0100	**Coarse-grain goal: Fractions as part of a whole**

Task description:

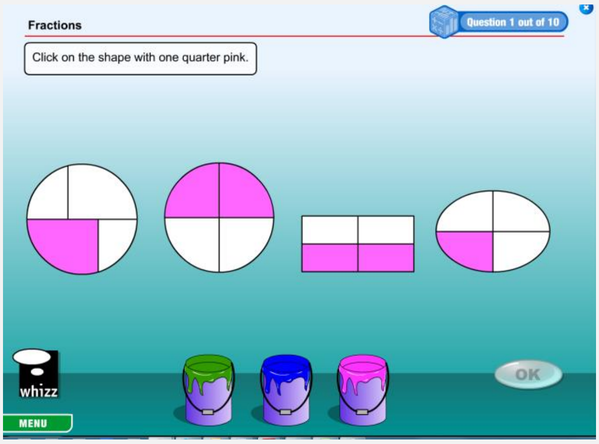


Task dimensions:

Fine-grain goal(s)	2. Interpret the size of a fractional part	3. Attribute fraction representation to symbol	4. Recognise different representations that are the same but look different	11. Identify the relationship between the size of the piece and the number of pieces		
Fraction Type	Set A	Set B	Set C	N/A		
		✓				
Interpretation	**Part-whole**	Ratio	Operator	Quotient	Measure	
Representation	All	**Area**	Number line	Sets	Liquid measur	Undefined
Task type	Procedural learning	Conceptual learning				
	Structured	Classify	Analyse	Interpret	Justify	Construct

Task code: MA_GBR_0700CAx0200	**Coarse-grain goal: Fractions as part of a whole**

6.1.1 Task description:

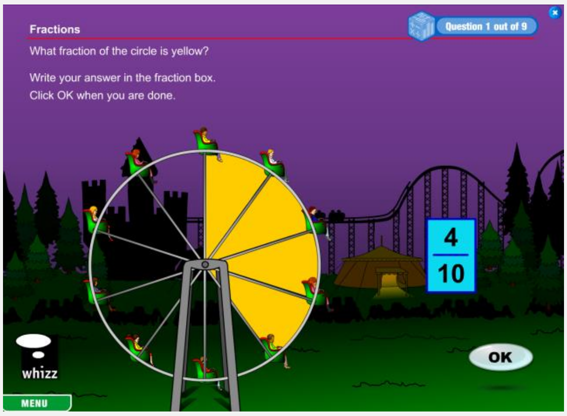


Task dimensions:

Fine-grain goal(s)	2. Interpret the size of a fractional part	4. Recognise different representations that are the same but look different				
Fraction Type	Set A ✓	Set B	Set C	N/A		
Interpretation	**Part-whole**	Ratio	Operator	Quotient	Measure	
Representation	All	**Area**	Number line	Sets	Liquid measur	Undefined
Task type	Procedural learning	Conceptual learning				
	Structured	Classify	Analyse	Interpret	Justify	Construct

Task code: MA_GBR_0825CAx0200	Coarse-grain goal: Fractions as part of a whole

Task description:



Task dimensions:

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<tr><td rowspan="2">Fraction Type</td><td>Set A</td><td>Set B</td><td colspan="2">Set C</td><td colspan="3">N/A</td></tr>
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<tr><td>Structured</td><td>Classify</td><td>Analyse</td><td>Interpret</td><td>Justify</td><td colspan="2">Construct</td></tr>
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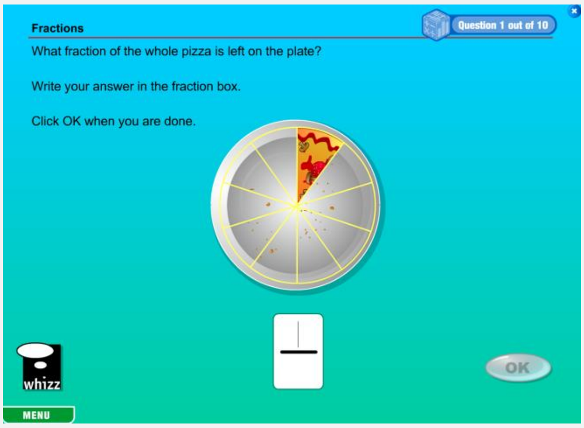
Task code: MA_GBR_0800CAx0200	**Coarse-grain goal: Fractions as part of a whole**
Task description:	

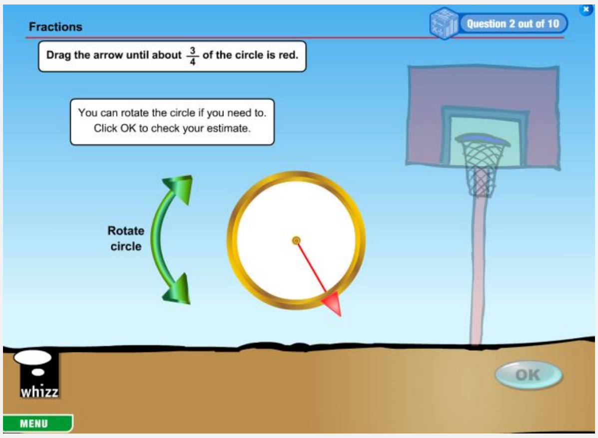


Task dimensions:

Fine-grain goal(s)	2. Interpret the size of a fractional part	3. Attribute fraction representation to symbol	4. Recognise different representations that are the same but look different	11. Identify the relationship between the size of the piece and the number of pieces		
Fraction Type	Set A	Set B	Set C	N/A		
	✓					
Interpretation	**Part-whole**	Ratio	Operator	Quotient	Measure	
Representation	All	**Area**	Number line	Sets	Liquid measur	Undefined
Task type	Procedural learning	Conceptual learning				
	Structured	Classify	Analyse	Interpret	Justify	Construct

Task code: MA_GBR_1300CAx0200	**Coarse-grain goal: Fractions as part of a whole**

Task description:

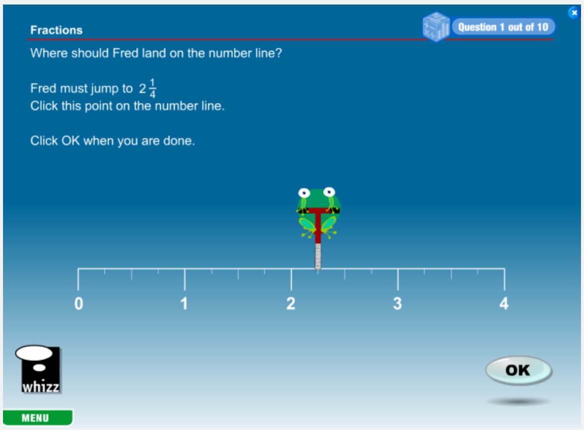


Task dimensions:

Fine-grain goal(s)	2. Interpret the size of a fractional part	3. Attribute fraction representation to symbol	11. Identify the relationship between the size of the piece and the number of pieces			
Fraction Type	Set A	Set B	Set C	N/A		
		✓				
Interpretation	**Part-whole**	Ratio	Operator	Quotient	Measure	
Representation	All	**Area**	Number line	Sets	Liquid measur	Undefined
Task type	Procedural learning	Conceptual learning				
	Structured	Classify	Analyse	Interpret	Justify	Construct

Task code: MA_GBR_8750CAx0100	**Coarse-grain goal: Fractions as part of a whole**

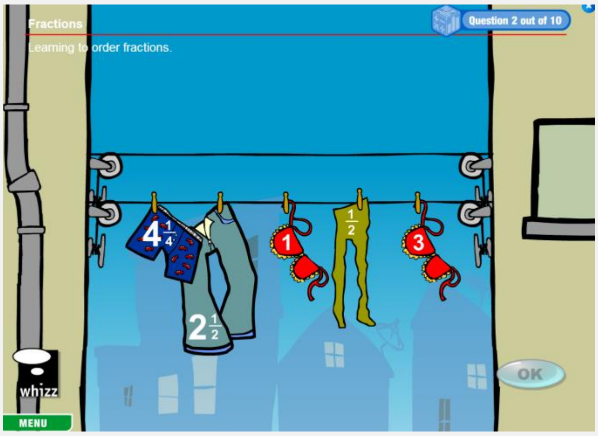
Task description:



Task dimensions:

Fine-grain goal(s)	2. Interpret the size of a fractional part	3. Attribute fraction representation to symbol				
Fraction Type	Set A	Set B	Set C	N/A		
			✓			
Interpretation	**Part-whole**	Ratio	Operator	Quotient	Measure	
Representation	All	Area	**Number line**	Sets	Liquid measures	Undefined
Task type	Procedural learning	Conceptual learning				
	Structured	Classify	Analyse	Interpret	Justify	Construct

Task code: MA_GBR_1025CAx0100	**Coarse-grain goal: Fractions as part of a whole**
Task description:	

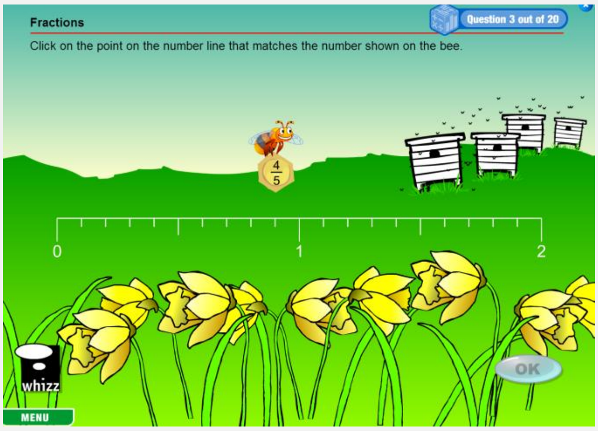


Task dimensions:

Fine-grain goal(s)	2. Interpret the size of a fractional part	3. Attribute fraction representation to symbol				
Fraction Type	Set A	Set B	Set C	N/A		
			✓			
Interpretation	**Part-whole**	Ratio	Operator	Quotient	Measure	
Representation	All	Area	**Number line**	Sets	Liquid measures	Undefined
Task type	Procedural learning	Conceptual learning				
	Structured	Classify	Analyse	Interpret	Justify	Construct

Task code: MA_GBR_1025CAx0200	**Coarse-grain goal: Fractions as part of a whole**

Task description:

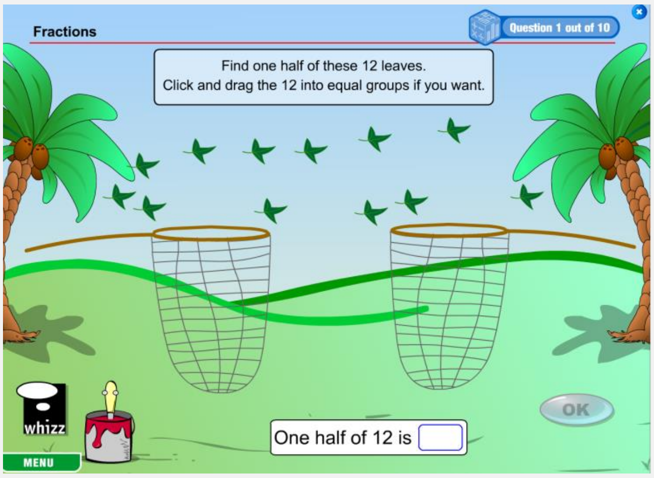


Task dimensions:

Fine-grain goal(s)	2. Interpret the size of a fractional part	3. Attribute fraction representation to symbol				
Fraction Type	Set A	Set B	Set C ✓	N/A		
Interpretation	**Part-whole**	Ratio	Operator	Quotient	Measure	
Representation	All	Area	**Number line**	Sets	Liquid measures	Undefined
Task type	Procedural learning	Conceptual learning				
	Structured	Classify	Analyse	Interpret	Justify	Construct

Task code: MA_GBR_0725CAx0100	**Coarse-grain goal: Fractions as part of a whole**

Task description:

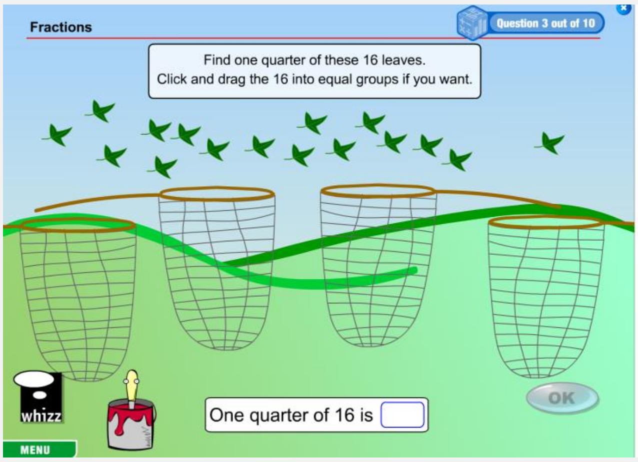


Task dimensions:

Fine-grain goal(s)	2. Interpret the size of a fractional part	3. Attribute fraction representation to symbol				
Fraction Type	Set A	Set B	Set C	N/A		
				✓		
Interpretation	**Part-whole**	Ratio	Operator	Quotient	Measure	
Representation	All	Area	Number line	**Sets**	Liquid measures	Undefined
Task type	Procedural learning	Conceptual learning				
	Structured	Classify	Analyse	Interpret	Justify	Construct

Task code: MA_GBR_0725CAx0200	**Coarse-grain goal: Fractions as part of a whole**

Task description:

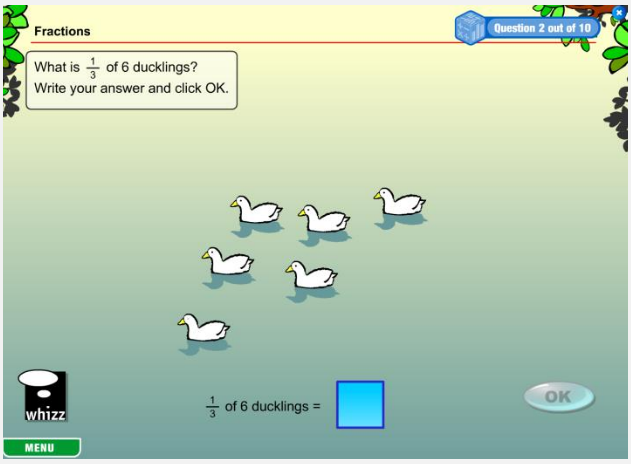


Task dimensions:

Fine-grain goal(s)	2. Interpret the size of a fractional part	3. Attribute fraction representation to symbol				
Fraction Type	Set A	Set B	Set C	N/A		
				✓		
Interpretation	**Part-whole**	Ratio	Operator	Quotient	Measure	
Representation	All	Area	Number line	**Sets**	Liquid measures	Undefined
Task type	Procedural learning	Conceptual learning				
	Structured	Classify	Analyse	Interpret	Justify	Construct

Task code: MA_GBR_0800CAx0300	**Coarse-grain goal: Fractions as part of a whole**

Task description:

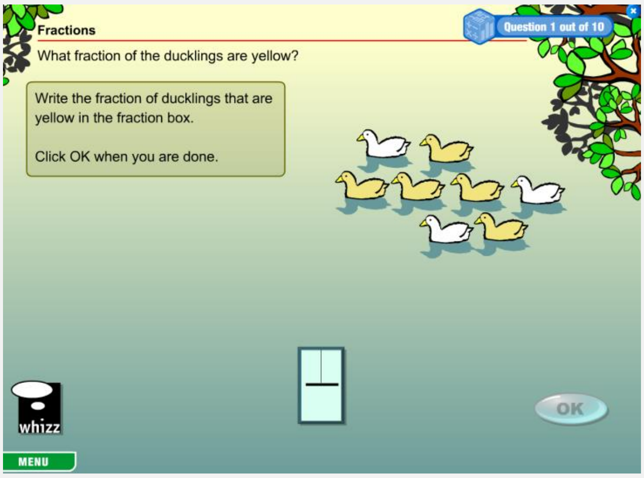


Task dimensions:

Fine-grain goal(s)	2. Interpret the size of a fractional part	3. Attribute fraction representation to symbol				
Fraction Type	Set A	Set B	Set C	N/A		
	✓					
Interpretation	**Part-whole**	Ratio	Operator	Quotient	Measure	
Representation	All	Area	Number line	**Sets**	Liquid measures	Undefined
Task type	Procedural learning	Conceptual learning				
	Structured	Classify	Analyse	Interpret	Justify	Construct

Task code: MA_GBR_0800CAx0400	**Coarse-grain goal: Fractions as part of a whole**

Task description:

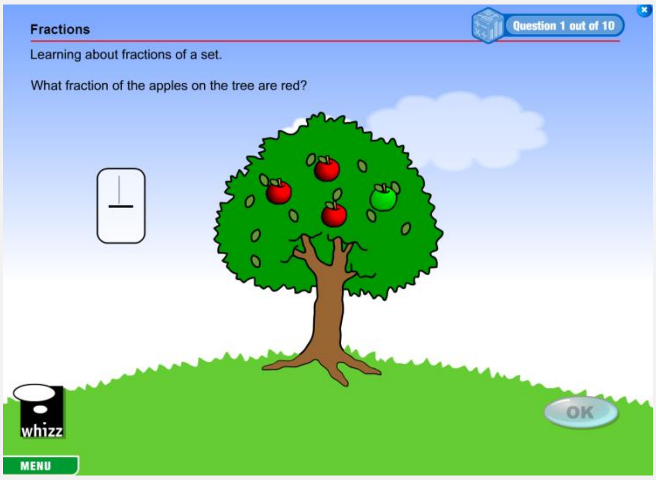


Task dimensions:

Fine-grain goal(s)	2. Interpret the size of a fractional part	3. Attribute fraction representation to symbol				
Fraction Type	Set A	Set B	Set C	N/A		
	✓					
Interpretation	**Part-whole**	Ratio	Operator	Quotient	Measure	
Representation	All	Area	Number line	**Sets**	Liquid measures	Undefined
Task type	Procedural learning	Conceptual learning				
	Structured	Classify	Analyse	Interpret	Justify	Construct

Task code: MA_GBR_0900CAx0200	Coarse-grain goal: Fractions as part of a whole

Task description:

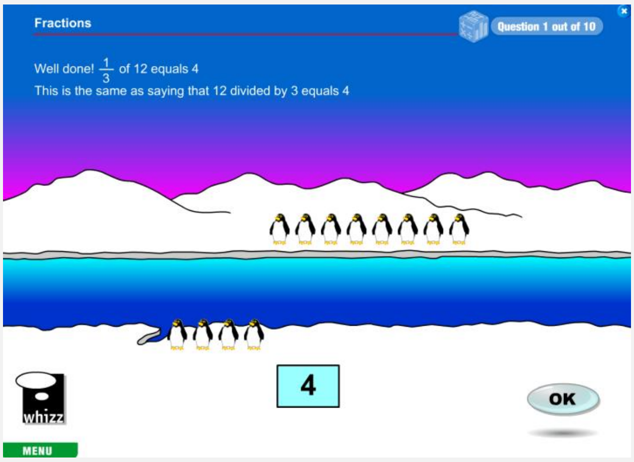


Task dimensions:

Fine-grain goal(s)	2. Interpret the size of a fractional part	3. Attribute fraction representation to symbol				
Fraction Type	Set A	Set B	Set C	N/A		
		✓				
Interpretation	**Part-whole**	Ratio	Operator	Quotient	Measure	
Representation	All	Area	Number line	**Sets**	Liquid measures	Undefined
Task type	Procedural learning	Conceptual learning				
	Structured	Classify	Analyse	Interpret	Justify	Construct

Task code: MA_GBR_0975CAx0200	**Coarse-grain goal: Fractions as part of a whole**

Task description:

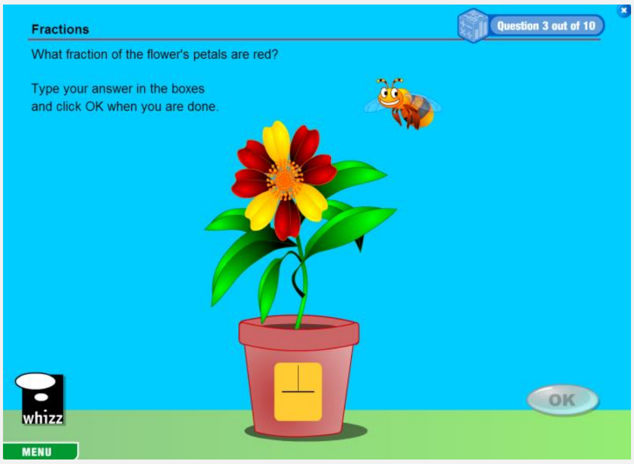


Task dimensions:

Fine-grain goal(s)	2. Interpret the size of a fractional part	3. Attribute fraction representation to symbol				
Fraction Type	Set A	Set B	Set C	N/A		
	✓					
Interpretation	**Part-whole**	Ratio	Operator	Quotient	Measure	
Representation	All	Area	Number line	**Sets**	Liquid measures	Undefined
Task type	Procedural learning	Conceptual learning				
	Structured	Classify	Analyse	Interpret	Justify	Construct

Task code: MA_GBR_0900CAx0100	**Coarse-grain goal: Fractions as part of a whole**

Task description:

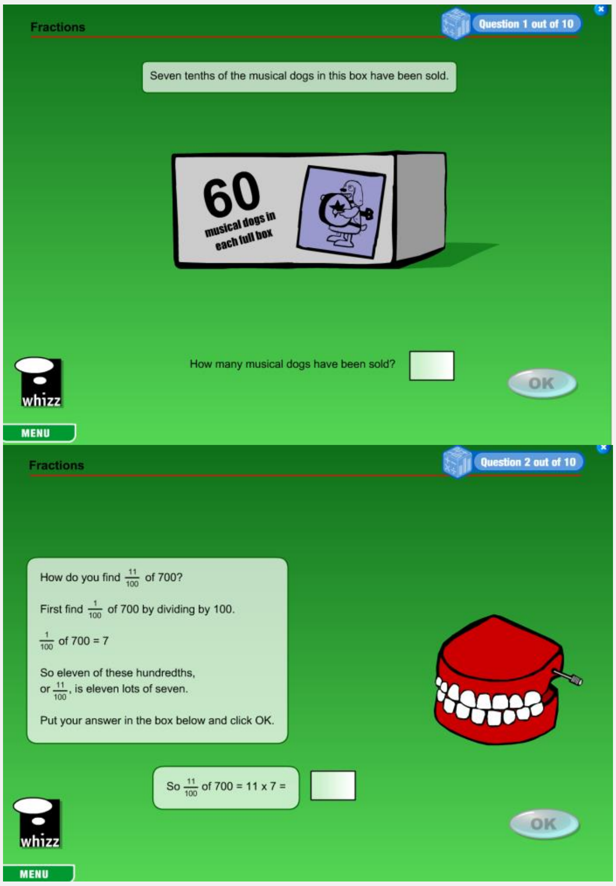


Task dimensions:

Fine-grain goal(s)	2. Interpret the size of a fractional part	3. Attribute fraction representation to symbol				
Fraction Type	Set A	Set B	Set C	N/A		
				✓		
Interpretation	**Part-whole**	Ratio	Operator	Quotient	Measure	
Representation	All	Area	Number line	**Sets**	Liquid measures	Undefined
Task type	Procedural learning	Conceptual learning				
	Structured	Classify	Analyse	Interpret	Justify	Construct

Task code: MA_GBR_1200CAx0500	Coarse-grain goal: Fractions as part of a whole

Task description:



Task dimensions:

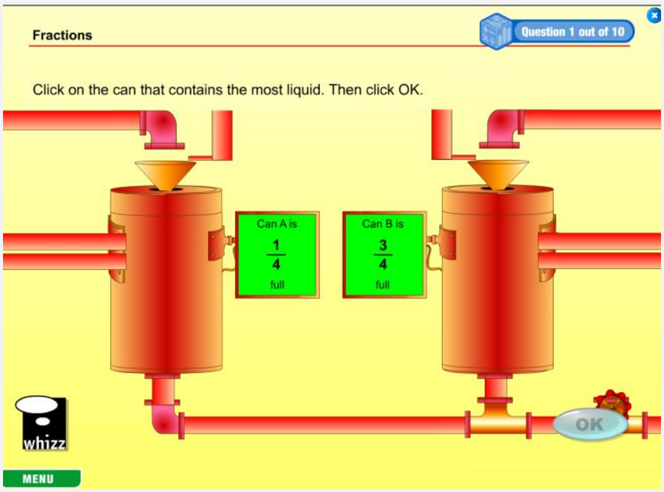
Fine-grain goal(s)	2. Interpret the size of a fractional part	3. Attribute fraction representation to symbol			

Fraction Type	Set A	Set B	Set C	N/A
				✓

Interpretation	**Part-whole**	Ratio	Operator	Quotient	Measure

Representation	All	Area	Number line	**Sets**	Liquid measures	Undefined
Task type	Procedural learning	Conceptual learning				
	Structured	Classify	Analyse	Interpret	Justify	Construct

Task code: MA_GBR_0875CAx0200	**Coarse-grain goal: Fractions as part of a whole**
Task description:	



Task dimensions:

Fine-grain goal(s)	2. Interpret the size of a fractional part	3. Attribute fraction representation to symbol	5. Compare unlike fractions	11. Identify the relationship between the size of the piece and the number of pieces	

Fraction Type	Set A	Set B	Set C	N/A
		✓		

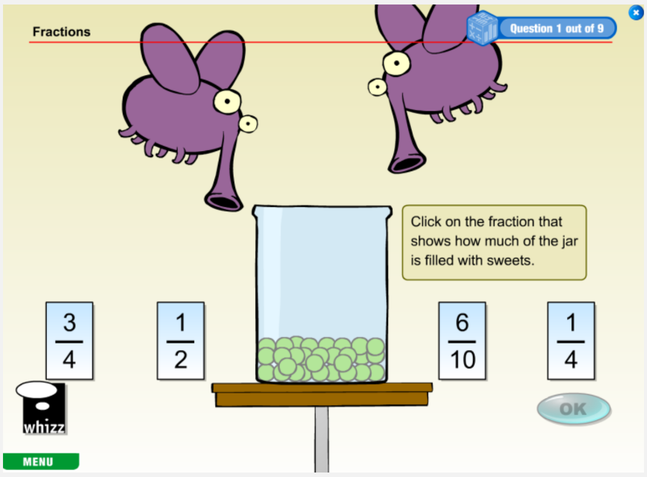
Interpretation	**Part-whole**	Ratio	Operator	Quotient	Measure

Representation	All	Area	Number line	Sets	**Liquid measure**	Undefined

Task type	Procedural learning	Conceptual learning				
	Structured	Classify	Analyse	Interpret	Justify	Construct

Task code: MA_GBR_0825CAx0500	**Coarse-grain goal: Fractions as part of a whole**

Task description:

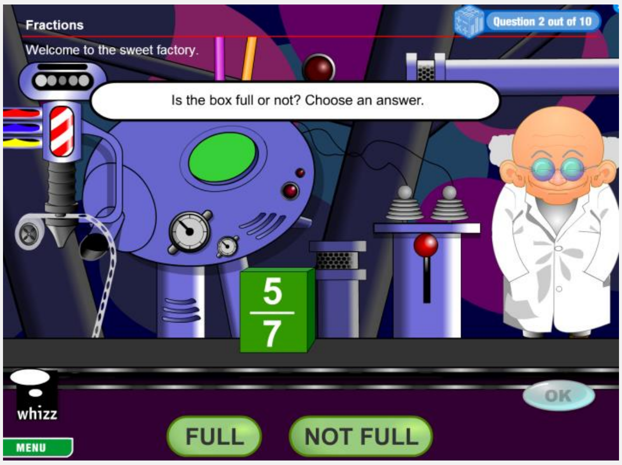


Task dimensions:

Fine-grain goal(s)	2. Interpret the size of a fractional part	3. Attribute fraction representation to symbol	11. Identify the relationship between the size of the piece and the number of pieces			
Fraction Type	Set A	Set B	Set C	N/A		
				✓		
Interpretation	**Part-whole**	Ratio	Operator	Quotient	Measure	
Representation	All	Area	Number line	Sets	**Liquid measur**	Undefined
Task type	Procedural learning	Conceptual learning				
	Structured	Classify	Analyse	Interpret	Justify	Construct

Task code: MA_GBR_0975CAx0100	**Coarse-grain goal: Fractions as part of a whole**

Task description:



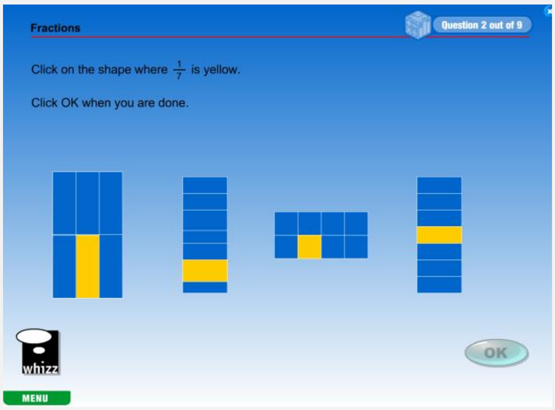
Task dimensions:

Fine-grain goal(s)	2. Interpret the size of a fractional part	3. Attribute fraction representation to symbol	5. Compare unlike fractions	11. Identify the relationship between the size of the piece and the number of pieces		
Fraction Type	Set A	Set B	Set C	N/A		
				✓		
Interpretation	**Part-whole**	Ratio	Operator	Quotient	Measure	
Representation	All	Area	Number line	Sets	**Liquid measur**	Undefined
Task type	Procedural learning	Conceptual learning				
	Structured	Classify	Analyse	Interpret	Justify	Construct

Version 2.0

Task code: MA_GBR_0800CAx0100	**Coarse-grain goal: Fractions as part of a whole**

Task description:

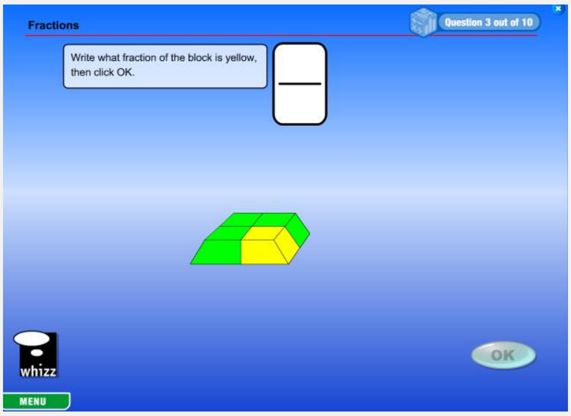


Task dimensions:

Fine-grain goal(s)	2. Interpret the size of a fractional part	3. Attribute fraction representation to symbol	4. Recognise different representations that are the same but look different			
Fraction Type	Set A	Set B	Set C	N/A		
				✓		
Interpretation	**Part-whole**	Ratio	Operator	Quotient	Measure	
Representation	All	**Area**	Number line	Sets	Liquid measur	Undefined
Task type	Procedural learning	Conceptual learning				
	Structured	Classify	Analyse	Interpret	Justify	Construct

Task code: MA_GBR_1200CAx0200	**Coarse-grain goal: Fractions as part of a whole**

Task description:



Task dimensions:

Fine-grain goal(s)	2. Interpret the size of a fractional part	3. Attribute fraction representation to symbol	4. Recognise different representations that are the same but look different			
Fraction Type	Set A	Set B	Set C	N/A		
		✓				
Interpretation	**Part-whole**	Ratio	Operator	Quotient	Measure	
Representation	All	**Area**	Number line	Sets	Liquid measur	Undefined
Task type	Procedural learning	Conceptual learning				
	Structured	Classify	Analyse	Interpret	Justify	Construct

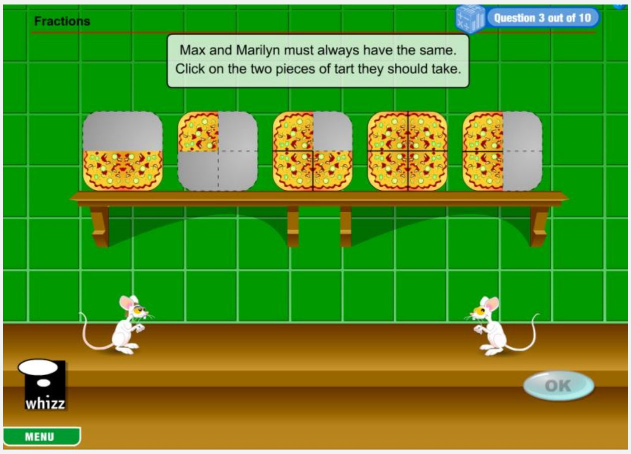
Task code: MA_GBR_1300CAx0100	**Coarse-grain goal: Fractions as part of a whole**

Task description:

Task dimensions:

Fine-grain goal(s)	2. Interpret the size of a fractional part	3. Attribute fraction representation to symbol	4. Recognise different representations that are the same but look different			
Fraction Type	Set A	Set B ✓	Set C	N/A		
Interpretation	**Part-whole**	Ratio	Operator	Quotient	Measure	
Representation	All	Area	Number line	Sets	Liquid measur	**Undefined**
Task type	Procedural learning	Conceptual learning				
	Structured	Classify	Analyse	Interpret	Justify	Construct

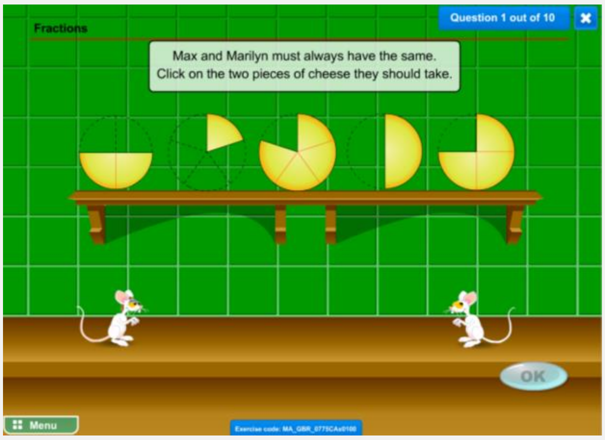
Task code: MA_GBR_0750CAx0100	**Coarse-grain goal: Equivalent fractions**
Task description:	



Task dimensions:

Fine-grain goal(s)	4. Recognise different representations that are the same but look different	5. Compare unlike fractions				
Fraction Type	Set A	Set B	Set C	N/A		
	✓					
Interpretation	**Part-whole**	Ratio	Operator	Quotient	Measure	
Representation	All	**Area**	Number line	Sets	Liquid measur	Undefined
Task type	Procedural learning	Conceptual learning				
	Structured	Classify	Analyse	Interpret	Justify	Construct

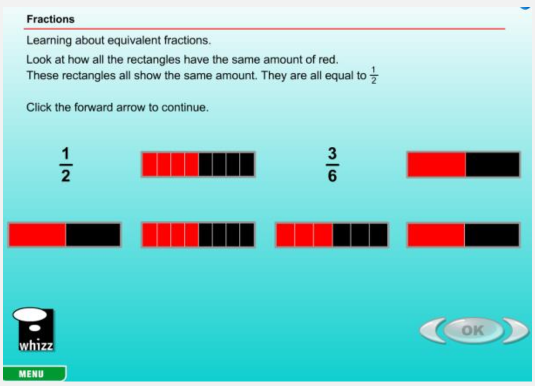
Task code: MA_GBR_0775CAx0100	**Coarse-grain goal: Equivalent fractions**					
Task description:						

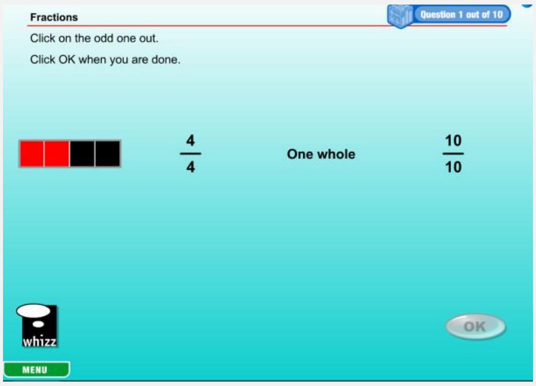


Task dimensions:

Fine-grain goal(s)	4. Recognise different representations that are the same but look different	5. Compare unlike fractions				
Fraction Type	Set A	Set B	Set C		N/A	
	✓					
Interpretation	**Part-whole**	Ratio	Operator	Quotient	Measure	
Representation	All	**Area**	Number line	Sets	Liquid measure	Undefined
Task type	Procedural learning	Conceptual learning				
	Structured	Classify	Analyse	Interpret	Justify	Construct

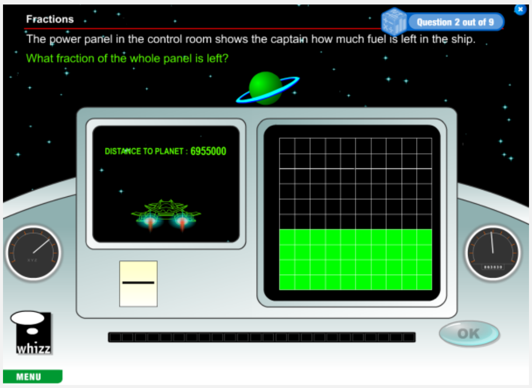
Task code: MA_GBR_0850CAx0100	**Coarse-grain goal: Equivalent fractions**					
Task description:						





Task dimensions:						
Fine-grain goal(s)	3. Attribute fraction representation to symbol	4. Recognise different representations that are the same but look different	5. Compare unlike fractions			
Fraction Type	Set A	Set B	Set C	N/A		
		✓				
Interpretation	**Part-whole**	Ratio	Operator	Quotient	Measure	
Representation	All	**Area**	Number line	Sets	Liquid measure	Undefined
Task type	Procedural learning	Conceptual learning				
	Structured	Classify	Analyse	Interpret	Justify	Construct

Task code: MA_GBR_1000CAx0200	**Coarse-grain goal: Equivalent fractions**
Task description:	

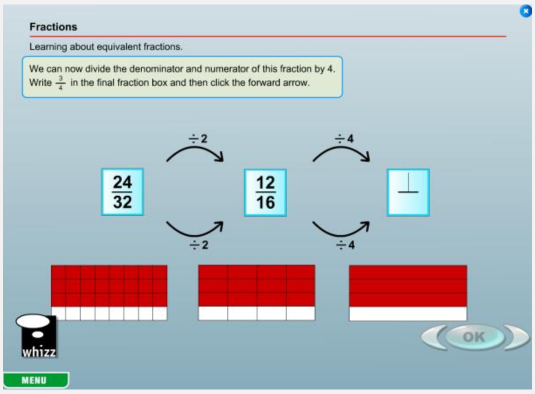


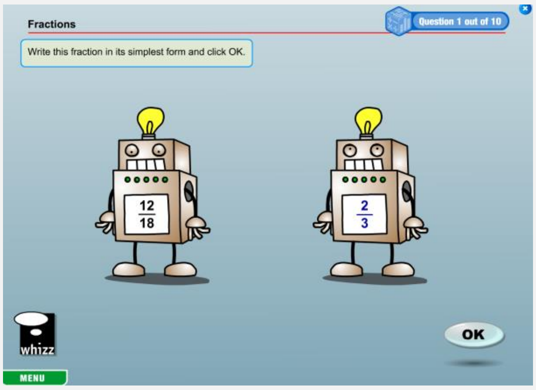
Task dimensions:

Fine-grain goal(s)	10. Cancel down to find equivalents	11. Identify the relationship between the size of the piece and the number of pieces				
Fraction Type	Set A	Set B	Set C	N/A		
				✓		
Interpretation	**Part-whole**	Ratio	Operator	Quotient	Measure	
Representation	All	**Area**	Number line	Sets	Liquid measures	Undefined
Task type	Procedural learning	Conceptual learning				
	Structured	Classify	Analyse	Interpret	Justify	Construct

Task code: MA_GBR_1125CAx0100	**Coarse-grain goal: Equivalent fractions**

Task description:



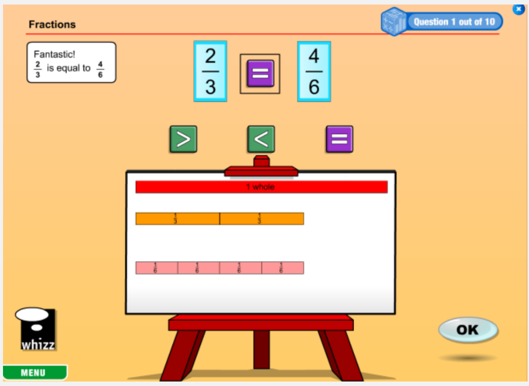


Task dimensions:

Fine-grain goal(s)	10. Cancel down to find equivalents	11. Identify the relationship between the size of the piece and the number of pieces				
Fraction Type	Set A	Set B	Set C	N/A		
				✓		
Interpretation	**Part-whole**	Ratio	Operator	Quotient	Measure	
Representation	All	**Area**	Number line	Sets	Liquid measure	Undefined
Task type	Procedural learning	Conceptual learning				
	Structured	Classify	Analyse	Interpret	Justify	Construct

Task code: MA_GBR_1225CAx0100	**Coarse-grain goal: Equivalent fractions**

Task description:

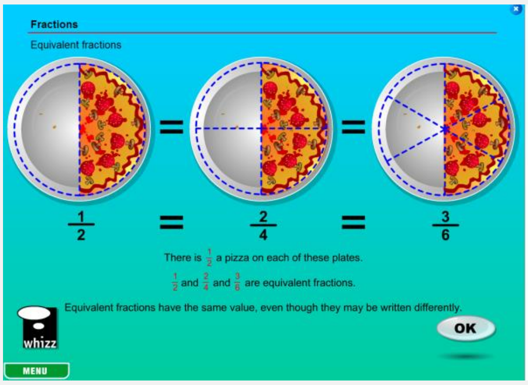


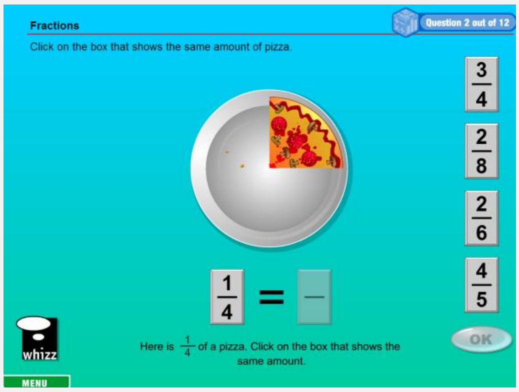
Task dimensions:

Fine-grain goal(s)	5. Compare unlike fractions	11. Identify the relationship between the size of the piece and the number of pieces				
Fraction Type	Set A	Set B	Set C	N/A		
		✓				
Interpretation	**Part-whole**	Ratio	Operator	Quotient	Measure	
Representation	All	**Area**	**Number line**	Sets	Liquid measure	Undefined
Task type	Procedural learning	Conceptual learning				
	Structured	Classify	Analyse	Interpret	Justify	Construct

Task code: MA_GBR_0950CAx0100	**Coarse-grain goal: Equivalent fractions**

Task description:



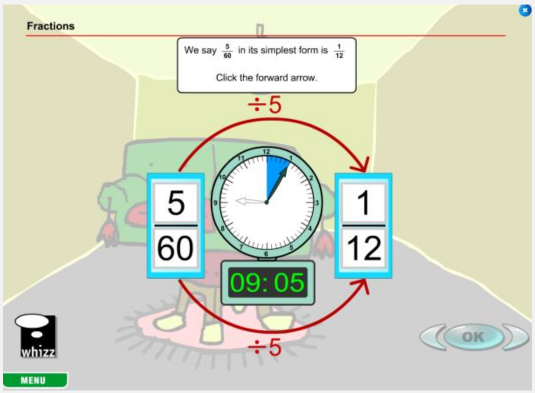


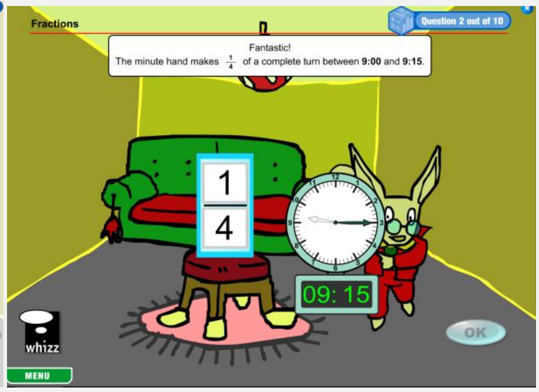
Task dimensions:

Fine-grain goal(s)	9. Partition to find equivalents					
Fraction Type	Set A	Set B	Set C	N/A		
		✓				
Interpretation	**Part-whole**	Ratio	Operator	Quotient	Measure	
Representation	All	**Area**	Number line	Sets	Liquid measures	Undefined
Task type	Procedural learning	Conceptual learning				
	Structured	Classify	Analyse	Interpret	Justify	Construct

Task code: MA_GBR_1200CAx0300	**Coarse-grain goal: Equivalent fractions**

Task description:



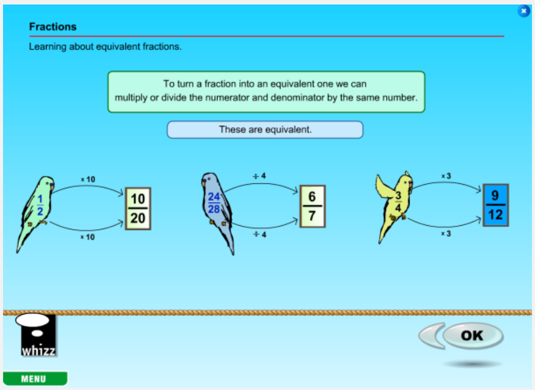


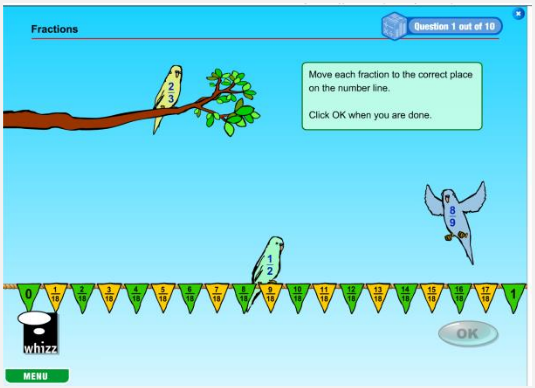
Task dimensions:

Fine-grain goal(s)	6. Expand fractions to find equivalents	7. Multiply numerator and denominator to find equivalents	10. Cancel down to find equivalents			
Fraction Type	Set A	Set B	Set C	N/A		
		✓				
Interpretation	**Part-whole**	Ratio	Operator	Quotient	Measure	
Representation	All	**Area**	Number line	Sets	Liquid measures	Undefined
Task type	Procedural learning	Conceptual learning				
	Structured	Classify	Analyse	Interpret	Justify	Construct

Task code: MA_GBR_1150CAx0300	**Coarse-grain goal: Equivalent fractions**

Task description:



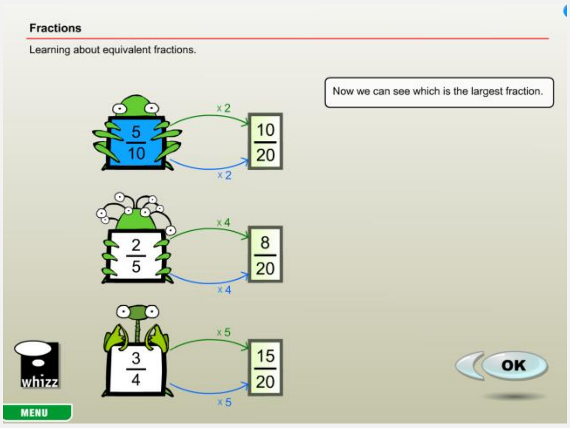


Task dimensions:

Fine-grain goal(s)	7. Multiply numerator and denominator to find equivalents	8. Make like denominators				
Fraction Type	Set A	Set B	Set C	N/A		
		✓				
Interpretation	**Part-whole**	Ratio	Operator	Quotient	Measure	
Representation	All	Area	**Number line**	Sets	Liquid measures	Undefined
Task type	Procedural learning	Conceptual learning				
	Structured	Classify	Analyse	Interpret	Justify	Construct

Task code: MA_GBR_1150CAx0100	**Coarse-grain goal: Equivalent fractions**

Task description:

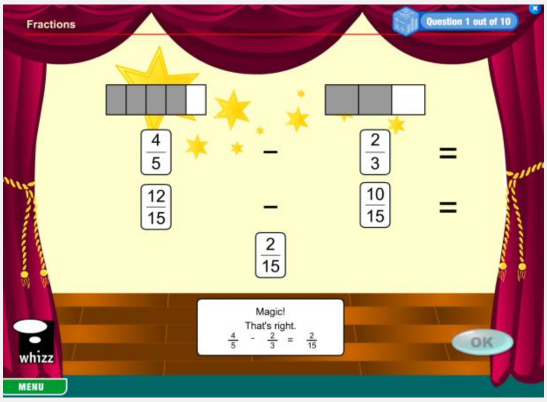


Task dimensions:

Fine-grain goal(s)	5. Compare unlike fractions	7. Multiply numerator and denominator to find equivalents	8. Make like denominators			
Fraction Type	Set A	Set B	Set C	N/A		
		✓				
Interpretation	**Part-whole**	Ratio	Operator	Quotient	Measure	
Representation	All	Area	Number line	Sets	Liquid measure	**Undefined**
Task type	Procedural learning	Conceptual learning				
	Structured	Classify	Analyse	Interpret	Justify	Construct

Task code: MA_GBR_1350CAx0200	**Coarse-grain goal: Add two fractions with the same denominator**

Task description:



Task dimensions:

Fine-grain goal(s)	12. Produce the sum of two fractions					
Fraction Type	Set A	Set B	Set C	N/A		
		✓				
Interpretation	**Part-whole**	Ratio	Operator	Quotient	Measure	
Representation	All	**Area**	Number line	Sets	Liquid measures	Undefined
Task type	Procedural learning	Conceptual learning				
	Structured	Classify	Analyse	Interpret	Justify	Construct

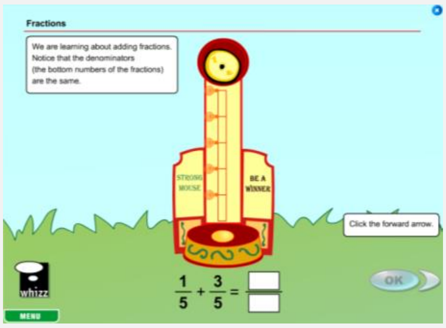
Task code: MA_GBR_0950CAx0100	**Coarse-grain goal: Add two fractions with the same denominator**

Task description:

Task dimensions:

Fine-grain goal(s)	12. Produce the sum of two fractions					
Fraction Type	Set A	Set B	Set C	N/A		
		✓				
Interpretation	**Part-whole**	Ratio	Operator	Quotient	Measure	
Representation	All	Area	**Number line**	Sets	Liquid measures	**Undefined**
Task type	Procedural learning	Conceptual learning				
	Structured	Classify	Analyse	Interpret	Justify	Construct

Task code: MA_GBR_0700CAx0100	**Coarse-grain goal: Add two fractions with the same denominator**
Task description:	

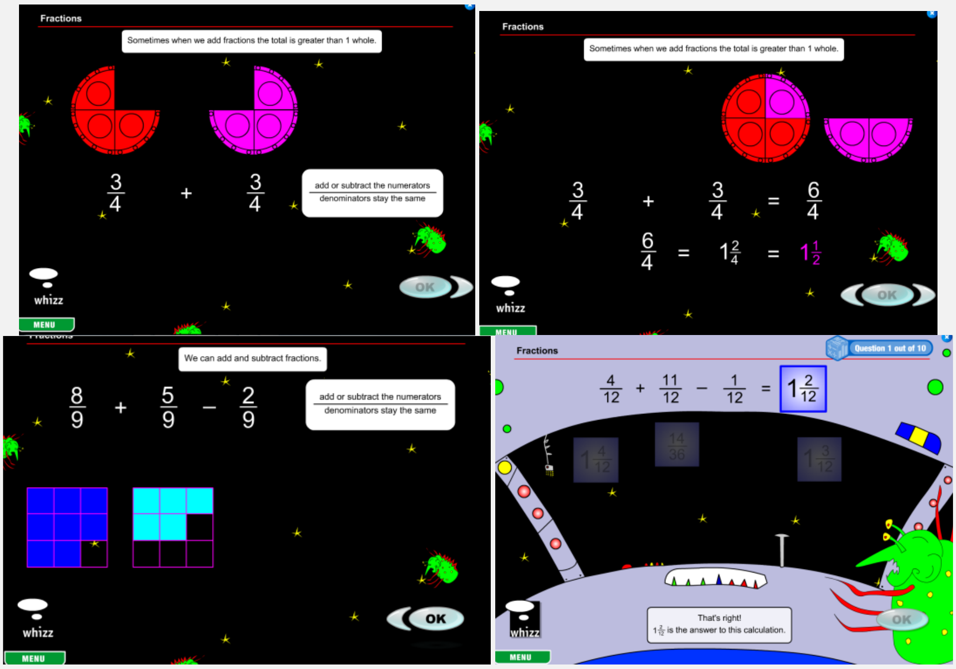


Task dimensions:

Fine-grain goal(s)	12. Produce the sum of two fractions					
Fraction Type	Set A	Set B	Set C	N/A		
		✓				
Interpretation	**Part-whole**	Ratio	Operator	Quotient	Measure	
Representation	All	Area	**Number line**	Sets	Liquid measures	Undefined
Task type	Procedural learning	Conceptual learning				
	Structured	Classify	Analyse	Interpret	Justify	Construct

Task code: MA_GBR_1275CAx0200	**Coarse-grain goal: Add two fractions with the same denominator**

Task description:

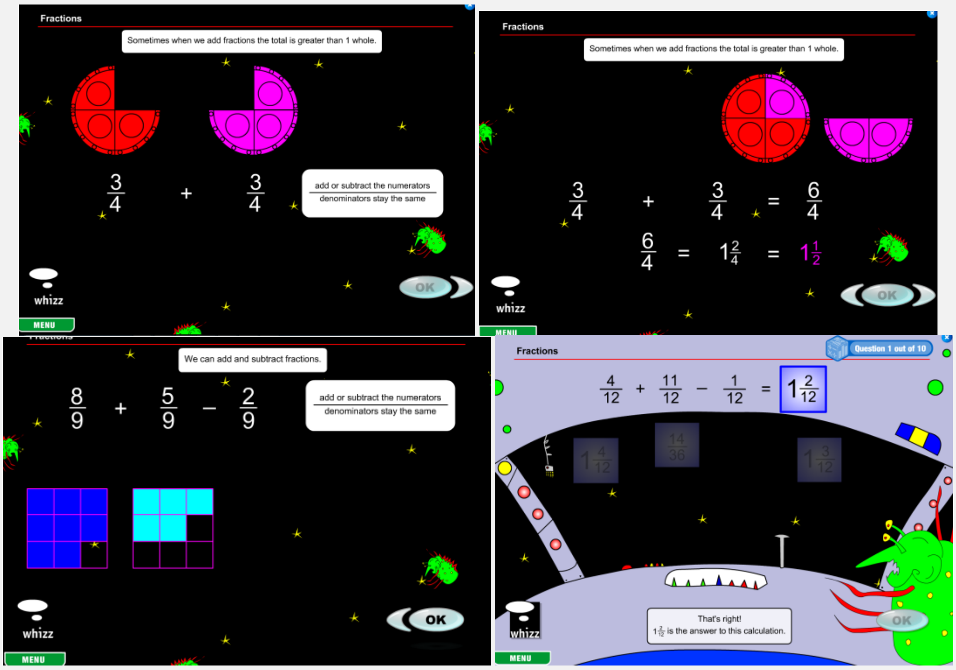


Task dimensions:

Fine-grain goal(s)	12. Produce the sum of two fractions					
Fraction Type	Set A	Set B	Set C	N/A		
			✓			
Interpretation	**Part-whole**	Ratio	Operator	Quotient	Measure	
Representation	All	**Area**	Number line	Sets	Liquid measures	Undefined
Task type	Procedural learning	Conceptual learning				
	Structured	Classify	Analyse	Interpret	Justify	Construct

Task code: MA_GBR_1275CAx0200	Coarse-grain goal: Subtract two fractions with the same denominator

Task description:



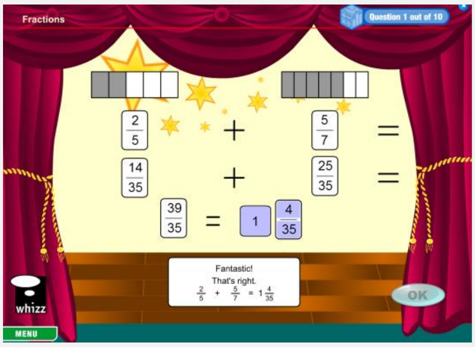
Task dimensions:

Fine-grain goal(s)	13. Produce the solution of subtracting two fractions					
Fraction Type	Set A	Set B	Set C	N/A		
			✓			
Interpretation	**Part-whole**	Ratio	Operator	Quotient	Measure	
Representation	All	**Area**	Number line	Sets	Liquid measures	Undefined

Task type	Procedural learning	Conceptual learning				
	Structured	Classify	Analyse	Interpret	Justify	Construct

Task code: MA_GBR_1350CAx0200	Coarse-grain goal: Add two fractions with denominators that are multiples of the same number

Task description:

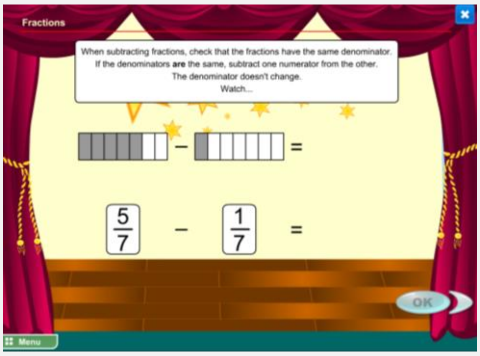


Task dimensions:

Fine-grain goal(s)	12. Produce the sum of two fractions					
Fraction Type	Set A	Set B	Set C	N/A		
			✓			
Interpretation	**Part-whole**	Ratio	Operator	Quotient	Measure	
Representation	All	**Area**	Number line	Sets	Liquid measures	Undefined
Task type	Procedural learning	Conceptual learning				
	Structured	Classify	Analyse	Interpret	Justify	Construct

Task code: MA_GBR_1275CAx0200	Coarse-grain goal: Subtract two fractions with denominators that are multiples of the same number

Task description:



Task dimensions:

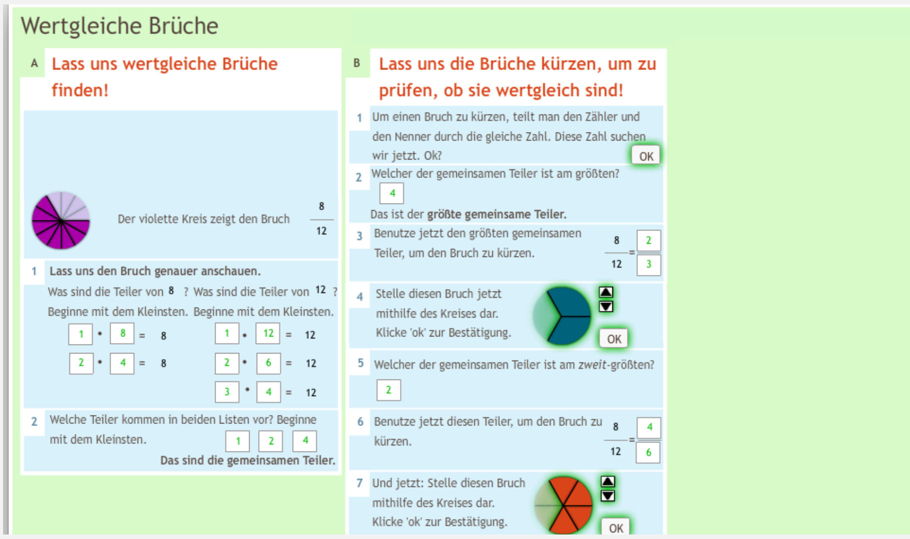
Fine-grain goal(s)	13. Produce the solution of subtracting two fractions					
Fraction Type	Set A	Set B	Set C	N/A		
			✓			
Interpretation	**Part-whole**	Ratio	Operator	Quotient	Measure	
Representation	All	**Area**	Number line	Sets	Liquid measures	Undefined
Task type	Procedural learning	Conceptual learning				
	Structured	Classify	Analyse	Interpret	Justify	Construct

Part 2: Fraction Tutor tasks

Task set I

Task code:	**Coarse-grain goal: Equivalent fractions**

Task description:



When students are challenged with the first set of task they are first asked to identify the factors of the denominator and numerator. By decomposing the factors students are prepared to determine the common factor of both the denominator and numerator. The common factors, in turn, help students to apply the procedure of reducing a fraction at hand.

There are 4 tasks available within this set.

Task dimensions:

Fine-grain goal(s)	6. Identify the factors of the numerator / denominator	7. Find the greatest common factor	12. Cancel down to find equivalence		
Fraction Type	Set A	Set B	Set C	N/A	
		✓			

Interpretation	**Part-whole**	Ratio	Operator	Quotient	Measure	
Representation	All	**Area**	Number line	Sets	Liquid measur	Undefined
Task type	Procedural learning	Conceptual learning				
	Structured	Classify	Analyse	Interpret	Justify	Construct

Task set II

Task code:	**Coarse-grain goal: Equivalent fractions**

Task description:

The second set of problem compares two fractions (fraction A and B) by reducing one of the fraction (fraction A) to its simplest form. In order to find the simplest form of fraction A, students need to find the common factors between the denominator and the numerator of this fraction. Assuming the to-be-reduced fraction is 6/16 students should first identify how 6 (and 16 respectively) can be split into a multiplication formula (i.e. 1-times 6, 2-times 3). By decomposing the denominator and numerator into factors students are enabled to identify the common factors of fraction A and are thus able to reduce fraction A by the identified common factor. The reduced form of fraction A in turn helps students to compare both fractions with each other.

In total we have 4 tasks of this set available.

Task dimensions:

Fine-grain goal(s)	6. Identify the factors of the numerator / denominator	7. Find the greatest common factor	12. Cancel down to find equivalence	5. Compare two fractions		
Fraction Type	Set A	Set B	Set C	N/A		
		✓				
Interpretation	**Part-whole**	Ratio	Operator	Quotient	Measure	
Representation	All	**Area**	Number line	Sets	Liquid measure	Undefined
Task type	Procedural learning	Conceptual learning				
	Structured	Classify	Analyse	Interpret	Justify	Construct

Task set III

Task code:	**Coarse-grain goal: Equivalent fractions**

Task description:

The third set of task focuses on expanding a given fractions with the factor 2,3,4 and 5. Students can do so by manipulating a representation of a fraction (e.g. a circle). As a second step, students are asked to express their expanding procedure with numeric symbols (e.g. 1/2 expanded with 2 becomes 2/4).

In total we have 4 tasks of this set available.

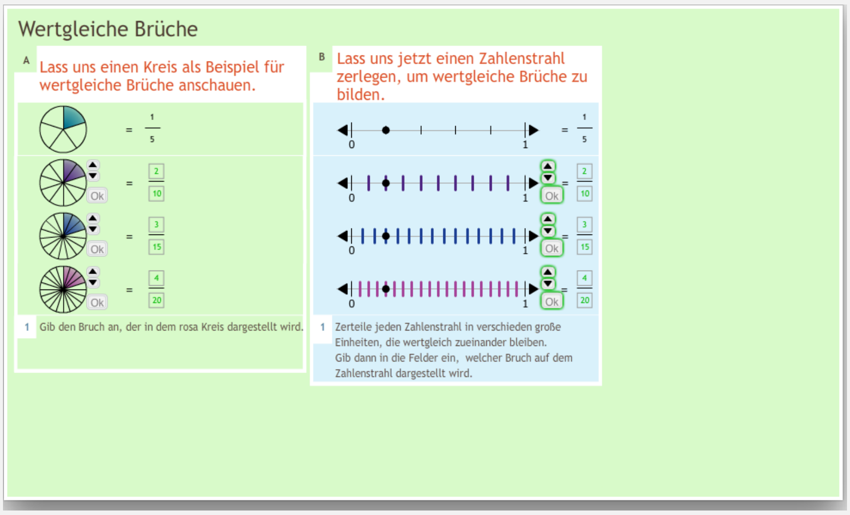
Task dimensions:

Fine-grain goal(s)	1. Interpret the size of a fractional part	8. Expand fractions to find equivalence	9. Multiply numerator and denominator to find equivalence	13. Identify the relationship between the size of the pieces and the number of pieces			
Fraction Type	Set A	Set B	Set C	N/A			
		✓					
Interpretation	**Part-whole**	Ratio	Operator	Quotient	Measure		
Representation	All	**Area**	Number line	Sets	Liquid measur	Undefined	
Task type	Procedural learning	Conceptual learning					
	Structured	Classify	Analyse	Interpret	Justify	Construct	

Task set IV (worked examples)

Task code:	**Coarse-grain goal: Equivalent fractions**

Task description:



The fourth sest of tasks focuses on expanding. The main instructional idea of this task set relies on a worked example: While the left representations (e.g. circles) is already expanded and aligned to the respective numeric symbol, the representations (e.g. number lines) on the right still needs to be expanded and in a second step aligned to the respective numbers.

In total we have 2 tasks of this set available.

Task dimensions:

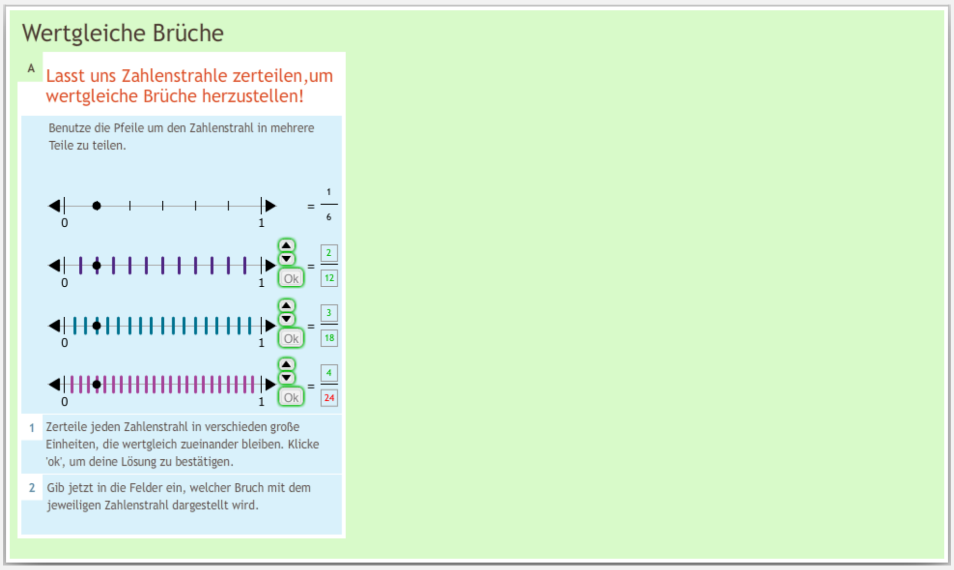
Fine-grain goal(s)	2. Interpret the size of a fractional part	3. Attribute fraction representation to a symbol	8. Expand fractions to find equivalence		
Fraction Type	Set A	Set B	Set C	N/A	
		✓			
Interpretation	**Part-whole**	Ratio	Operator	Quotient	Measure

Representation	All	**Area**	**Number line**	Sets	Liquid measur	Undefin ed
Task type	Procedural learning	Conceptual learning				
	Structured	Classify	Analyse	Interpr et	Justify	Constru ct

Task set V

Task code:	**Coarse-grain goal: Equivalent fractions**

Task description:



Concerning the fifth set of problem, students are encouraged to partition a single graphical representation (e.g. number line) and to name the "produced" fraction.

In total we have 4 tasks of this set available.

Task dimensions:

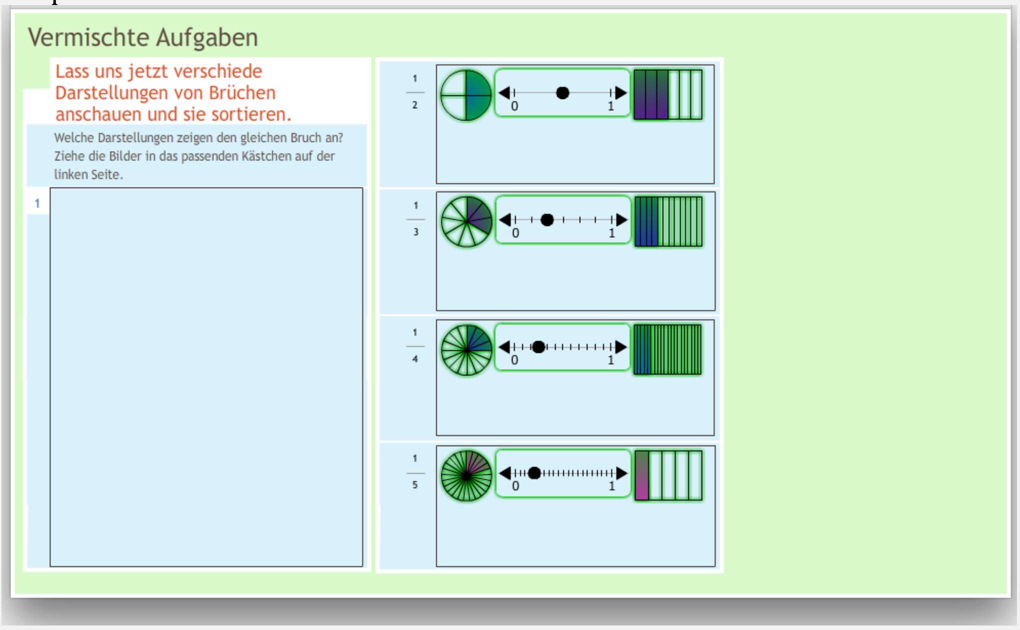
Fine-grain goal(s)	2. Interpret the size of a fractional part	3. Attribute fraction representation to a symbol	8. Expand fractions to find equivalence			
Fraction Type	Set A	Set B	Set C	N/A		
		✓				
Interpretation	**Part-whole**	Ratio	Operator	Quotient	Measure	
Representation	All	Area	**Number line**	Sets	Liquid measur	Undefined

Task type	Procedural learning	Conceptual learning				
	Structured	Classify	Analyse	Interpret	Justify	Construct

Task set VI

Task code:	**Coarse-grain goal: Equivalent fractions**

Task description:



The last set of tasks asks students to align different representations of fractions to a numerical value. Indeed, students are asked to drag and drop a circle, a number line and a rectangle expressing the same fraction differently into the same box (right side).

In total we have 2 tasks of this category available.

Task dimensions:

Fine-grain goal(s)	3. Attribute fraction representation to a symbol	13. Identify the relationship between the size of the pieces and the number of pieces			

Fraction Type	Set A	Set B	Set C	N/A
		✓		

Interpretation	**Part-whole**	Ratio		Operator	Quotient	Measure
Representation	All	**Area**	**Number line**	Sets	Liquid measur	Undefined
Task type	Procedural learning	Conceptual learning				
	Structured	Classify	Analyse	Interpret	Justify	Construct

Appendix IV: Exploratory Tasks

Section 1 provides an overview of the exploratory tasks with the task variations. Section 2 presents an example of an exploratory task from each coarse-grain goal.

1. Overview of exploratory tasks

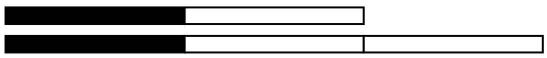
In the table below the exploratory tasks are presented. The task variations are shown below each task description. Set A, Set B, Set C refer to the fraction types. The fraction representations are shown with A=Area, NL=Number Line, S= Sets, LM = Liquid Measures, Und = Undefined [student's own choice], All = All reps used concurrently.

<table>
<tr><th rowspan="2">CGG</th><th rowspan="2">Task number</th><th rowspan="2" colspan="6">Task description</th><th colspan="5">Task type</th><th rowspan="2">No. of tasks</th></tr>
<tr><th>Classify</th><th>Analyse</th><th>Interpret</th><th>Justify</th><th>Construct</th></tr>
<tr><td rowspan="5">0</td><td rowspan="5">1</td><td colspan="6">Think of a fraction. Make it using each of the representations.</td><td rowspan="5"></td><td rowspan="5"></td><td rowspan="5">X</td><td rowspan="5"></td><td rowspan="5"></td><td rowspan="5">1</td></tr>
<tr><td colspan="2">Set A</td><td colspan="2">Set B</td><td colspan="2">Set C</td></tr>
<tr><td colspan="2">N/A</td><td colspan="2">N/A</td><td colspan="2">N/A</td></tr>
<tr><td>All</td><td>A</td><td>NL</td><td>S</td><td>LM</td><td>Und</td></tr>
<tr><td>X</td><td></td><td></td><td></td><td></td><td></td></tr>
<tr><td rowspan="5">0</td><td rowspan="5">2</td><td colspan="6">Now use the partition tool to partition each fraction into 2, 3, 4 and then 5.</td><td rowspan="5"></td><td rowspan="5"></td><td rowspan="5">X</td><td rowspan="5"></td><td rowspan="5"></td><td rowspan="5">1</td></tr>
<tr><td colspan="2">Set A</td><td colspan="2">Set B</td><td colspan="2">Set C</td></tr>
<tr><td colspan="2">N/A</td><td colspan="2">N/A</td><td colspan="2">N/A</td></tr>
<tr><td>All</td><td>A</td><td>NL</td><td>S</td><td>LM</td><td>Und</td></tr>
<tr><td>X</td><td></td><td></td><td></td><td></td><td></td></tr>
</table>

<table>
<tr><td>0</td><td>3</td><td>Make a fraction and then copy it. Change the copied fraction into a different representation.
<table>
<tr><td colspan="2">Set A</td><td colspan="2">Set B</td><td colspan="2">Set C</td></tr>
<tr><td colspan="2">N/A</td><td colspan="2">N/A</td><td colspan="2">N/A</td></tr>
<tr><td>All</td><td>A</td><td>NL</td><td>S</td><td>LM</td><td>Und</td></tr>
<tr><td></td><td></td><td></td><td></td><td></td><td>X</td></tr>
</table></td><td></td><td></td><td>X</td><td></td><td></td><td>1</td></tr>
<tr><td>0</td><td>4</td><td>Make a fraction. Change its colour.
<table>
<tr><td colspan="2">Set A</td><td colspan="2">Set B</td><td colspan="2">Set C</td></tr>
<tr><td colspan="2">N/A</td><td colspan="2">N/A</td><td colspan="2">N/A</td></tr>
<tr><td>All</td><td>A</td><td>NL</td><td>S</td><td>LM</td><td>Und</td></tr>
<tr><td></td><td></td><td></td><td></td><td></td><td>X</td></tr>
</table></td><td>X</td><td></td><td></td><td></td><td></td><td>1</td></tr>
<tr><td>0</td><td>5</td><td>Show a/b + x/y = c/d.
<table>
<tr><td colspan="2">Set A</td><td colspan="2">Set B</td><td colspan="2">Set C</td></tr>
<tr><td colspan="2">a/b = 1/5
x/y = 2/5
c/d = 3/5</td><td colspan="2">a/b = 2/9
x/y = 3/9
c/d = 5/9</td><td colspan="2">a/b = 6/7
x/y = 5/7
c/d = 11/7</td></tr>
<tr><td>All</td><td>A</td><td>NL</td><td>S</td><td>LM</td><td>Und</td></tr>
<tr><td></td><td></td><td></td><td></td><td></td><td>X</td></tr>
</table></td><td></td><td></td><td></td><td></td><td>X</td><td>3 sets x 1 rep = 3</td></tr>
<tr><td>0</td><td>6</td><td>Show a/b - x/y = c/d.
<table>
<tr><td colspan="2">Set A</td><td colspan="2">Set B</td><td colspan="2">Set C</td></tr>
<tr><td colspan="2">a/b = 4/6
x/y = 1/6
c/d = 3/6</td><td colspan="2">a/b = 4/7
x/y = 2/7
c/d = 2/7</td><td colspan="2">a/b = 13/6
x/y = 8/6
c/d = 5/6</td></tr>
<tr><td>All</td><td>A</td><td>NL</td><td>S</td><td>LM</td><td>Und</td></tr>
<tr><td></td><td></td><td></td><td></td><td></td><td>X</td></tr>
</table></td><td></td><td></td><td></td><td></td><td>X</td><td>3 sets x 1 rep = 3</td></tr>
</table>

0	7	**"The numerator of a fraction is smaller than the denominator." Show why this is always true, sometimes true or never true.**									X		1
		Set A		*Set B*		*Set C*							
		N/A		N/A		N/A							
		All	*A*	*NL*	*S*	*LM*	*Und*						
							X						
								1	0	3	1	6	

<table>
<tr><td>1</td><td>1</td><td>Show whether a/b is bigger or smaller than x/y.
<table>
<tr><th colspan="2">Set A</th><th colspan="2">Set B</th><th colspan="2">Set C</th></tr>
<tr><td colspan="2">a/b = 1/2
x/y = 1/4</td><td colspan="2">a/b = 1/3
x/y = 1/5</td><td colspan="2">a/b = 1/10
x/y = 1/12</td></tr>
<tr><th>All</th><th>A</th><th>NL</th><th>S</th><th>LM</th><th>Und</th></tr>
<tr><td></td><td></td><td></td><td></td><td></td><td>X</td></tr>
</table></td><td></td><td></td><td></td><td></td><td>X</td><td>3 sets x 1 rep = 3</td></tr>
<tr><td>1</td><td>2</td><td>Is a/b greater than or less than x/y? [REP]
<table>
<tr><th colspan="2">Set A</th><th colspan="2">Set B</th><th colspan="2">Set C</th></tr>
<tr><td colspan="2">a/b = 2/3
x/y = 3/5</td><td colspan="2">a/b = 3/4
x/y = 5/7</td><td colspan="2">a/b = 18/8
x/y = 12/5</td></tr>
<tr><th>All</th><th>A</th><th>NL</th><th>S</th><th>LM</th><th>Und</th></tr>
<tr><td></td><td>Use rectangles to work it out.</td><td>Use number lines to work it out.</td><td>Use sets to work it out.</td><td>Use liquids to work it out.</td><td></td></tr>
</table></td><td></td><td></td><td></td><td></td><td>X</td><td>3 sets x 4 reps = 12</td></tr>
<tr><td>1</td><td>3</td><td>Amelia says, "a/b is bigger than x/y because b is bigger than y." Show what you think.
<table>
<tr><th colspan="2">Set A</th><th colspan="2">Set B</th><th colspan="2">Set C</th></tr>
<tr><td colspan="2">a/b = 1/8
x/y = 1/3</td><td colspan="2">a/b = 1/10
x/y = 1/8</td><td colspan="2">a/b = 16/10
x/y = 13/8</td></tr>
<tr><th>All</th><th>A</th><th>NL</th><th>S</th><th>LM</th><th>Und</th></tr>
<tr><td></td><td></td><td></td><td></td><td></td><td>X</td></tr>
</table></td><td></td><td>X</td><td></td><td></td><td></td><td>3 sets x 1 rep = 3</td></tr>
</table>

<table>
<tr>
<td>1</td><td>4</td>
<td>Georgia says, "a/b is bigger than x/y because a is bigger than x." Use Fractions Lab to show what you think.
<table>
<tr><th colspan="2">Set A</th><th colspan="2">Set B</th><th colspan="2">Set C</th></tr>
<tr><td colspan="2">a/b = 5/10
x/y = 4/10</td><td colspan="2">a/b = 6/8
x/y = 2/3</td><td colspan="2">a/b = 7/10
x/y = 5/8</td></tr>
<tr><th>All</th><th>A</th><th>NL</th><th>S</th><th>LM</th><th>Und</th></tr>
<tr><td></td><td></td><td></td><td></td><td></td><td>X</td></tr>
</table>
</td>
<td></td><td>X</td><td></td><td></td><td></td><td>3 sets x 1 rep = 3</td>
</tr>
<tr>
<td>1</td><td>5</td>
<td>Shaun said, "1/2 = 1/3" and drew this picture.

<table>
<tr><th colspan="2">Set A</th><th colspan="2">Set B</th><th colspan="2">Set C</th></tr>
<tr><td colspan="2">a/b = 1/2
x/y = 1/3</td><td colspan="2">a/b = 1/4
x/y = 1/5</td><td colspan="2">a/b = 2/5
x/y = 3/7</td></tr>
<tr><th>All</th><th>A</th><th>NL</th><th>S</th><th>LM</th><th>Und</th></tr>
<tr><td></td><td>X</td><td></td><td></td><td></td><td></td></tr>
</table>
</td>
<td></td><td>X</td><td></td><td></td><td></td><td>3 sets x 1 rep = 3</td>
</tr>
<tr>
<td>1</td><td>6</td>
<td>"A fraction is always smaller than 1." Show why this is always true, sometimes true or never true.
<table>
<tr><th colspan="2">Set A</th><th colspan="2">Set B</th><th colspan="2">Set C</th></tr>
<tr><td colspan="2">N/A</td><td colspan="2">N/A</td><td colspan="2">N/A</td></tr>
<tr><th>All</th><th>A</th><th>NL</th><th>S</th><th>LM</th><th>Und</th></tr>
<tr><td></td><td></td><td></td><td></td><td></td><td>X</td></tr>
</table>
</td>
<td></td><td></td><td></td><td>X</td><td></td><td>1</td>
</tr>
</table>

<table>
<tr><td rowspan="5">1</td><td rowspan="5">7</td><td colspan="6">Which is bigger? a/b of n or x/y of s?</td><td rowspan="5"></td><td rowspan="5"></td><td rowspan="5"></td><td rowspan="5">X</td><td rowspan="5"></td><td rowspan="5">3 sets x 1 rep = 3</td></tr>
<tr><td colspan="2">Set A</td><td colspan="2">Set B</td><td colspan="2">Set C</td></tr>
<tr><td colspan="2">a/b = 1/6

n = 24

x/y = 1/7

s = 21</td><td colspan="2">a/b = 2/3

n =27

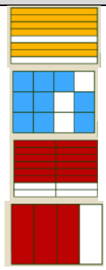
x/y = 3/4

s = 20</td><td colspan="2">a/b = 5/4

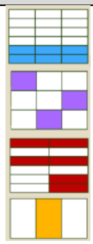
n = 52

x/y = 7/3

s = 27</td></tr>
<tr><td>All</td><td>A</td><td>NL</td><td>S</td><td>LM</td><td>Und</td></tr>
<tr><td></td><td></td><td></td><td>X</td><td></td><td></td></tr>
<tr><td></td><td></td><td></td><td></td><td></td><td></td><td></td><td></td><td>0</td><td>9</td><td>0</td><td>4</td><td>15</td><td></td></tr>
</table>

<table>
<tr><td rowspan="5">2</td><td rowspan="5">1</td><td colspan="6">What do you notice about these fractions? [REP]</td><td rowspan="5">X</td><td rowspan="5"></td><td rowspan="5"></td><td rowspan="5"></td><td rowspan="5"></td><td rowspan="5">3 sets x 3 reps = 12</td></tr>
<tr><td colspan="2">Set A</td><td colspan="2">Set B</td><td colspan="2">Set C</td></tr>
<tr><td colspan="2">[Show four fractions = 3/4]</td><td colspan="2">[Show four fractions = 3/8]</td><td colspan="2">Show four fractions = 8/5]</td></tr>
<tr><td>All</td><td>A</td><td>NL</td><td>S</td><td>LM</td><td>Und</td></tr>
<tr><td></td><td></td><td>[4 number line examples]</td><td>[4 sets examples]</td><td>[4 liquid measures examples]</td><td></td></tr>
<tr><td rowspan="5">2</td><td rowspan="5">2</td><td colspan="6">Make a fraction that is equivalent to a/b.</td><td rowspan="5"></td><td rowspan="5"></td><td rowspan="5"></td><td rowspan="5"></td><td rowspan="5">X</td><td rowspan="5">3 sets x 1 rep = 3</td></tr>
<tr><td colspan="2">Set A</td><td colspan="2">Set B</td><td colspan="2">Set C</td></tr>
<tr><td colspan="2">a/b = 1/2</td><td colspan="2">a/b = 1/5</td><td colspan="2">a/b = 1/6</td></tr>
<tr><td>All</td><td>A</td><td>NL</td><td>S</td><td>LM</td><td>Und</td></tr>
<tr><td></td><td></td><td></td><td></td><td></td><td>X</td></tr>
<tr><td rowspan="5">2</td><td rowspan="5">3</td><td colspan="6">Use each representation to show a/b.</td><td rowspan="5"></td><td rowspan="5"></td><td rowspan="5">X</td><td rowspan="5"></td><td rowspan="5"></td><td rowspan="5">3 sets x 1 rep = 3</td></tr>
<tr><td colspan="2">Set A</td><td colspan="2">Set B</td><td colspan="2">Set C</td></tr>
<tr><td colspan="2">2/3</td><td colspan="2">4/5</td><td colspan="2">7/8</td></tr>
<tr><td>All</td><td>A</td><td>NL</td><td>S</td><td>LM</td><td>Und</td></tr>
<tr><td>X</td><td></td><td></td><td></td><td></td><td></td></tr>
</table>

<table>
<tr><td rowspan="5">2</td><td rowspan="5">4</td><td colspan="6">[NAME] looks at these fractions and says they are all different.

[insert three [REP] fractions that are equivalent but look different]</td><td rowspan="5"></td><td rowspan="5">X</td><td rowspan="5"></td><td rowspan="5"></td><td rowspan="5"></td><td rowspan="5">3 sets x 4 reps = 12</td></tr>
<tr><td colspan="2">Set A</td><td colspan="2">Set B</td><td colspan="2">Set C</td></tr>
<tr><td colspan="2">1/2; 2/4; 4/8; 6/12</td><td colspan="2">1/3; 3/9; 4/12; 6/18</td><td colspan="2">5/3; 10/6; 25/15; 30/18</td></tr>
<tr><td>All</td><td>A</td><td>NL</td><td>S</td><td>LM</td><td>Und</td></tr>
<tr><td></td><td>Martha</td><td>Simon</td><td>Elise</td><td>George</td><td></td></tr>
<tr><td rowspan="5">2</td><td rowspan="5">5</td><td colspan="6">Which fraction is the odd one out? [REP]</td><td rowspan="5">X</td><td rowspan="5"></td><td rowspan="5"></td><td rowspan="5"></td><td rowspan="5"></td><td rowspan="5">3 sets x 4 reps = 12</td></tr>
<tr><td colspan="2">Set A</td><td colspan="2">Set B</td><td colspan="2">Set C</td></tr>
<tr><td colspan="2">1/3; 6/18; 6/12; 3/9</td><td colspan="2">2/3; 4/6; 8/12; 12/15</td><td colspan="2">4/3; 8/6; 11/9; 16/12</td></tr>
<tr><td>All</td><td>A</td><td>NL</td><td>S</td><td>LM</td><td>Und</td></tr>
<tr><td></td><td></td><td>[4 number line examples]</td><td>[4 sets examples]</td><td>[4 liquid measures examples]</td><td></td></tr>
<tr><td rowspan="5">2</td><td rowspan="5">6</td><td colspan="6">[NAME] says "a/b = x/y because a times b equals y". Show why you agree or disagree.</td><td rowspan="5"></td><td rowspan="5">X</td><td rowspan="5"></td><td rowspan="5"></td><td rowspan="5"></td><td rowspan="5">3 sets x 1 rep = 3</td></tr>
<tr><td colspan="2">Set A</td><td colspan="2">Set B</td><td colspan="2">Set C</td></tr>
<tr><td colspan="2">[Michel]

a/b = 3/4

x/y = 1/12</td><td colspan="2">[Sam]

a/b = 2/5

x/y = 1/10</td><td colspan="2">[Amir]

a/b = 7/3

x/y = 1/21</td></tr>
<tr><td>All</td><td>A</td><td>NL</td><td>S</td><td>LM</td><td>Und</td></tr>
<tr><td></td><td></td><td></td><td></td><td></td><td>X</td></tr>
</table>

2	7	**Make a/b using [REP]. Use the partition tool to partition it.**								X			3 sets x 4 reps = 12
		Set A		*Set B*		*Set C*							
		a/b = 3/4		a/b = 5/8		a/b = 7/5							
		All	*A*	*NL*	*S*	*LM*	*Und*						
			a rectangle	a number line	sets	liquid measures							
2	8	**Make a fraction that equals a/b and has c as denominator.**										X	3 sets x 1 rep = 3
		Set A		*Set B*		*Set C*							
		a/b = 1/6 c = 18		a/b = 3/4 c = 12		a/b = 7/3 c = 12							
		All	*A*	*NL*	*S*	*LM*	*Und*						
							X						
								24	15	15	0	6	

<table>
<tr><td>3a+</td><td>1</td><td>Show how you could make this fraction by adding two fractions. [Show fraction using the REP]
<table>
<tr><td colspan="2">Set A</td><td colspan="2">Set B</td><td colspan="2">Set C</td></tr>
<tr><td colspan="2">3/5</td><td colspan="2">4/7</td><td colspan="2">12/9</td></tr>
<tr><td>All</td><td>A</td><td>NL</td><td>S</td><td>LM</td><td>Und</td></tr>
<tr><td>X</td><td></td><td></td><td></td><td></td><td></td></tr>
</table></td><td></td><td></td><td></td><td></td><td>X</td><td>3 sets x 4 reps = 12</td></tr>
<tr><td>3a-</td><td>1</td><td>Show a subtraction where the solution is a/b.
<table>
<tr><td colspan="2">Set A</td><td colspan="2">Set B</td><td colspan="2">Set C</td></tr>
<tr><td colspan="2">a/b = 2/5</td><td colspan="2">a/b = 4/15</td><td colspan="2">a/b = 5/9</td></tr>
<tr><td>All</td><td>A</td><td>NL</td><td>S</td><td>LM</td><td>Und</td></tr>
<tr><td></td><td></td><td></td><td></td><td></td><td>X</td></tr>
</table></td><td></td><td></td><td></td><td></td><td>X</td><td>3 sets x 1 rep = 3</td></tr>
<tr><td>3a-</td><td>2</td><td>[NAME] poured out a/b and had x/y left. Show how full the jug was before he began.
<table>
<tr><td colspan="2">Set A</td><td colspan="2">Set B</td><td colspan="2">Set C</td></tr>
<tr><td colspan="2">[George]
a/b = 7/10
x/y = 1/10</td><td colspan="2">[James]
a/b = 4/8
x/y = 3/8</td><td colspan="2">[Matt]
a/b = 8/7
x/y = 5/7</td></tr>
<tr><td>All</td><td>A</td><td>NL</td><td>S</td><td>LM</td><td>Und</td></tr>
<tr><td></td><td></td><td></td><td></td><td>X</td><td></td></tr>
</table></td><td></td><td></td><td></td><td></td><td>X</td><td>3 sets x 1 rep = 3</td></tr>
<tr><td></td><td></td><td></td><td>0</td><td>0</td><td>0</td><td>0</td><td>6</td><td></td></tr>
</table>

<table>
<tr><td rowspan="5">3b+</td><td rowspan="5">1</td><td colspan="6">[NAME] used [REP] to add a/b and x/y. Can you find out what her answer was?</td><td rowspan="5"></td><td rowspan="5"></td><td rowspan="5"></td><td rowspan="5"></td><td rowspan="5">X</td><td rowspan="5">3 sets x 4 reps = 12</td></tr>
<tr><td colspan="2">Set A</td><td colspan="2">Set B</td><td colspan="2">Set C</td></tr>
<tr><td colspan="2">a/b = 1/6
x/y = 5/12</td><td colspan="2">a/b = 2/3
x/y = 2/9</td><td colspan="2">a/b = 4/3
x/y = 3/6</td></tr>
<tr><td>All</td><td>A</td><td>NL</td><td>S</td><td>LM</td><td>Und</td></tr>
<tr><td></td><td>April
rectangles</td><td>Clara
Number lines</td><td>June
sets</td><td>Mary
liquid measures</td><td></td></tr>
<tr><td rowspan="5">3b+</td><td rowspan="5">2</td><td colspan="6">Show how you could make this fraction by adding two fractions with different denominators.</td><td rowspan="5"></td><td rowspan="5"></td><td rowspan="5"></td><td rowspan="5"></td><td rowspan="5">X</td><td rowspan="5">3 sets x 4 reps = 12</td></tr>
<tr><td colspan="2">Set A</td><td colspan="2">Set B</td><td colspan="2">Set C</td></tr>
<tr><td colspan="2">7/12</td><td colspan="2">12/18</td><td colspan="2">16/12</td></tr>
<tr><td>All</td><td>A</td><td>NL</td><td>S</td><td>LM</td><td>Und</td></tr>
<tr><td></td><td></td><td></td><td></td><td></td><td></td></tr>
</table>

3b-	1	**Show a subtraction where the solution is a/b. The denominators should be multiples of the same number.**												X	3 sets x 1 rep = 3
		Set A		*Set B*		*Set C*									
		a/b = 1/2		a/b = 2/6		a/b = 9/5									
		All	*A*	*NL*	*S*	*LM*	*Und*								
							X								
3b-	2	**[NAME] poured a/b out. He had x/y left. How much was in the jug before he began?**												X	3 sets x 1 rep = 3
		Set A		*Set B*		*Set C*									
		[Erik] a/b = 6/10 x/y = 1/5		[Jon] a/b = 4/8 x/y = 1/4		[William] a/b = 14/12 x/y = 2/3									
		All	*A*	*NL*	*S*	*LM*	*Und*								
						X									
										0	0	0	0	30	

<table>
<tr><td rowspan="5">3c+</td><td rowspan="5">1</td><td colspan="6">[NAME] used [REP] to think about adding fractions. Her answer is shown here. One of her fractions was a/b. Can you make it too?</td><td rowspan="5"></td><td rowspan="5"></td><td rowspan="5"></td><td rowspan="5"></td><td rowspan="5">X</td><td rowspan="5">3 sets x 4 reps = 12</td></tr>
<tr><td colspan="2">Set A</td><td colspan="2">Set B</td><td colspan="2">Set C</td></tr>
<tr><td colspan="2">a/b = 1/4 (for answer 7/12)</td><td colspan="2">a/b = 1/3 (for answer 7/12)</td><td colspan="2">a/b = 7/12 (for answer 4/3)</td></tr>
<tr><td>All</td><td>A</td><td>NL</td><td>S</td><td>LM</td><td>Und</td></tr>
<tr><td></td><td>Isra</td><td>Aster</td><td>Jess</td><td>Sara</td><td></td></tr>
<tr><td rowspan="5">3c-</td><td rowspan="5">1</td><td colspan="6">[NAME] is using [REP] to think about subtracting fractions. He wants to work out a/b - c/d. Can you show him?</td><td rowspan="5"></td><td rowspan="5"></td><td rowspan="5"></td><td rowspan="5"></td><td rowspan="5">X</td><td rowspan="5">3 sets x 4 reps = 12</td></tr>
<tr><td colspan="2">Set A</td><td colspan="2">Set B</td><td colspan="2">Set C</td></tr>
<tr><td colspan="2">a/b = 2/3
c/d = 1/2</td><td colspan="2">a/b = 6/8
c/d = 1/5</td><td colspan="2">a/b = 10/4
c/d = 6/5</td></tr>
<tr><td>All</td><td>A</td><td>NL</td><td>S</td><td>LM</td><td>Und</td></tr>
<tr><td></td><td>Zach</td><td>Rohan</td><td>Elliott</td><td>Jack</td><td></td></tr>
<tr><td></td><td></td><td></td><td></td><td></td><td></td><td></td><td></td><td>0</td><td>0</td><td>0</td><td>0</td><td>24</td><td></td></tr>
</table>

2. Selection of exploratory tasks from each coarse-grain goal

<table>
<tr><td>Task code: 0.1</td><td colspan="6">Coarse-grain goal: Familiarisation</td></tr>
<tr><td colspan="7">6.1.2 Task description: “Think of a fraction. Make it using each of the representations.”</td></tr>
<tr><td colspan="7">6.1.3 Task dimensions:</td></tr>
<tr><td>Fine-grain goal(s)</td><td>1. Recognise the whole</td><td>2. Interpret the size of a fractional part</td><td>4. Recognise different representations that are the same but look different</td><td></td><td></td><td></td></tr>
<tr><td rowspan="2">Fraction Type</td><td>Set A</td><td>Set B</td><td>Set C</td><td colspan="3">N/A</td></tr>
<tr><td></td><td></td><td></td><td colspan="3">✓</td></tr>
<tr><td>Interpretation</td><td>Part-whole</td><td>Ratio</td><td>Operator</td><td>Quotient</td><td colspan="2">Measure</td></tr>
<tr><td>Representation</td><td>All</td><td>Area</td><td>Number line</td><td>Sets</td><td>Liquid measures</td><td>Undefined</td></tr>
<tr><td rowspan="2">Task type</td><td>Procedural learning</td><td colspan="5">Conceptual learning</td></tr>
<tr><td>Structured</td><td>Classify</td><td>Analyse</td><td>Interpret</td><td>Justify</td><td>Construct</td></tr>
<tr><td colspan="7">Expected student behaviour:
The student will choose a fraction they are familiar with and make it using each of the representations available to them (five in final version including symbol). They will notice how the fraction is represented using the different fractions. They may note that they have not seen particular representations before.</td></tr>
<tr><td colspan="7">Example difficulties and opportunities:
• Student makes 1/2: Encourage to make a more difficult / interesting fraction.
• Student has used some but not all of the representations: Encourage to keep going.</td></tr>
<tr><td colspan="7">Task completion:
Student has completed task when five representations of the same fraction have been created.</td></tr>
<tr><td rowspan="2">Final Reflective prompt:</td><td colspan="3">GENERIC</td><td colspan="3">SPECIFIC</td></tr>
<tr><td colspan="3">How are the representations you have used the same and how are they different?</td><td colspan="3">Which representation did you find most challenging to use? Why?</td></tr>
<tr><td>Task-specific vocabulary:</td><td colspan="2">• rectangle, area
• number line, line</td><td colspan="2">• set, objects, stars/circles [or whatever object is in the set]
• fraction, number, symbol</td><td colspan="2">• jug, measuring jug, liquid measure/s, liquid, juice, water
• numerator, denominator</td></tr>
</table>

Misconceptions:	GLOBAL • Student may discuss the symbol as two whole numbers (E.g. "x over y" without acknowledging that the fraction is part of a whole)	SITUATED • Student may think that a fraction can only be represented in one or two ways (e.g. symbol and rectangle). • Student may not know that the denominator represents the number of equal divisions (e.g. may think that they making ¼ but they make 1/5 because they are making one part shaded and then four further parts to make the "quarter") • Student may not know that the numerator represents the part of the whole (student may try to make 2/1 instead of ½)

Task code: 1.1	**Coarse-grain goal: Fractions as part of a whole**					
6.1.4 Task description: Show whether a/b is bigger or smaller than x/y.						
6.1.5 Task dimensions:						
Fine-grain goal(s)	2. Interpret the size of a fractional part	4. Recognise different representations that are the same but look different	13. Identify the relationship between the size of the piece and the number of pieces			
Fraction Type	Set A a/b = 1/2 x/y = 1/4	Set B a/b = 1/3 x/y = 1/5	Set C a/b = 1/10 x/y = 1/12	N/A		
Interpretation	**Part-whole**	Ratio	Operator	Quotient	Measure	
Representation	All	Area	Number line	Sets	Liquid measures	**Undefined**
Task type	Procedural learning	Conceptual learning				
	Structured	Classify	Analyse	Interpret	Justify	**Construct**
Expected student behaviour: Student will make two fractions using the same representation, one is a/b and the other x/y. The student will compare the two fractions visually, perhaps by placing them on top of each other, beside each other or overlapping them. They will note which fractional part is larger.						
Example difficulties and opportunities: • Student makes two fractions. One is a/b+1 and the other is x/y+1: Ask student to check the fractions by looking at the symbols. • Student uses preferred representation: Encourage student to use a different representation.						
Task completion: Student will have made two fractions with the same representation. One fraction will be a/b and the other fraction will be x/y. The student will have used the comparison box to compare them.						
Final Reflective prompt:	GENERIC How are the representations you have used the same and how are they different?	SPECIFIC Why did you choose the representation you used? How does this help you to think about fractions?				

Task-specific vocabulary:	• one half, one quarter • one third, one fifth • one tenth, one twelfth • denominator • bigger, smaller, because	
Misconceptions:	GLOBAL • Student may think that a/b is smaller than x/y because b is smaller than y (i.e. treating denominator as whole number). • Student may make a/b+1 and x/y+1 to show a:b ratio and x:y ratio instead of a/b and x/y.	SITUATED •

<table>
<tr><td>Task code: 2.6</td><td colspan="6">Coarse-grain goal: Equivalent fractions</td></tr>
<tr><td colspan="7">6.1.6 Task description: [NAME] says "a/b = x/y because a times b equals y". Show on the screen why you agree or disagree.</td></tr>
<tr><td colspan="7">6.1.7 Task dimensions:</td></tr>
<tr><td>Fine-grain goal(s)</td><td>3. Attribute fraction representation to symbol</td><td>8. Generate a common denominator</td><td>11. Partition to find equivalents</td><td></td><td colspan="2"></td></tr>
<tr><td rowspan="2">Fraction Type</td><td>Set A</td><td>Set B</td><td>Set C</td><td colspan="3">N/A</td></tr>
<tr><td>[Michel]
a/b = 3/4
x/y = 1/12</td><td>[Sam]
a/b = 2/5
x/y = 1/10</td><td>[Amir]
a/b = 7/3
x/y = 1/21</td><td colspan="3"></td></tr>
<tr><td>Interpretation</td><td>Part-whole</td><td>Ratio</td><td>Operator</td><td>Quotient</td><td colspan="2">Measure</td></tr>
<tr><td>Representation</td><td>All</td><td>Area</td><td>Number line</td><td>Sets</td><td>Liquid measures</td><td>Undefined</td></tr>
<tr><td rowspan="2">Task type</td><td>Procedural learning</td><td colspan="5">Conceptual learning</td></tr>
<tr><td>Structured</td><td>Classify</td><td>Analyse</td><td>Interpret</td><td>Justify</td><td>Construct</td></tr>
<tr><td colspan="7">Expected student behaviour:
The student will make two fractions (a/b and x/y) and compare them using the comparison box.</td></tr>
<tr><td colspan="7">Example difficulties and opportunities:
• Student makes a/b and x/y but uses two different representations: Explain why it is important to select the same representation to be able to compare more effectively</td></tr>
<tr><td colspan="7">Task completion:
Student will have made two fractions (3/4 and 1/12) and compared them with the comparison box.</td></tr>
<tr><td>Final Reflective prompt:</td><td colspan="3">GENERIC
Do you agree or disagree with [NAME]? Why? And please remember to use mathematical words.</td><td colspan="3">SPECIFIC
Why did you choose to show your thinking in the way you did here?</td></tr>
<tr><td>Task-specific vocabulary:</td><td colspan="6">• equal • equivalent •</td></tr>
<tr><td>Misconceptions:</td><td colspan="3">GLOBAL
• Treating numerator/denominator as whole numbers</td><td colspan="3">SITUATED
• Student agrees that Michel is correct</td></tr>
</table>

<table>
<tr><td>Task code: 3a+.1</td><td colspan="6">Coarse-grain goal: Add two fractions with the same denominator</td></tr>
<tr><td colspan="7">6.1.8 Task description: Show how you could make this fraction by adding two fractions. [Show fraction using the REP]</td></tr>
<tr><td colspan="7">6.1.9 Task dimensions:</td></tr>
<tr><td>Fine-grain goal(s)</td><td>14. Produce the sum of two fractions</td><td></td><td></td><td></td><td colspan="2"></td></tr>
<tr><td rowspan="2">Fraction Type</td><td>Set A</td><td>Set B</td><td>Set C</td><td colspan="3">N/A</td></tr>
<tr><td>3/5</td><td>4/7</td><td>12/9</td><td colspan="3"></td></tr>
<tr><td>Interpretation</td><td>Part-whole</td><td>Ratio</td><td>Operator</td><td>Quotient</td><td colspan="2">Measure</td></tr>
<tr><td>Representation</td><td>All</td><td>Area</td><td>Number line</td><td>Sets</td><td>Liquid measures</td><td>Undefined</td></tr>
<tr><td rowspan="2">Task type</td><td>Procedural learning</td><td colspan="5">Conceptual learning</td></tr>
<tr><td>Structured</td><td>Classify</td><td>Analyse</td><td>Interpret</td><td>Justify</td><td>Construct</td></tr>
<tr><td colspan="7">Expected student behaviour:
The student will make two fractions from the same representation of their choice. The two fractions will share the same denominator.</td></tr>
<tr><td colspan="7">Example difficulties and opportunities:
• Student uses preferred representation: Encourage student to use a different representation.
• Student makes two fractions where the denominators, if added together as whole numbers, would equal the denominator required. E.g. 2/3 + 1/2 = 3/5: Ask student to check their answer, encouraging them to check the denominators.</td></tr>
<tr><td colspan="7">Task completion:
The student will drag the two fractions into the addition box to check their answer is correct.</td></tr>
<tr><td>Final Reflective prompt:</td><td colspan="3">GENERIC
Can you please explain why you chose to complete the task in that way?</td><td colspan="3">SPECIFIC
What have you learnt about adding fractions?</td></tr>
<tr><td>Task-specific vocabulary:</td><td colspan="6">• add • equal • denominator • equals • numerator • same</td></tr>
<tr><td>Misconceptions:</td><td colspan="3">GLOBAL
• Treating numerator / denominator as whole numbers</td><td colspan="3">SITUATED
• Adding across denominators</td></tr>
</table>

Task code: 3a-.2	**Coarse-grain goal: Subtract two fractions with the same denominator**					
6.1.10 Task description: [NAME] poured out a/b and had x/y left. Show how full the jug was before he began.						
6.1.11 Task dimensions:						
Fine-grain goal(s)	15. Produce the solution of subtracting two fractions					
Fraction Type	Set A	Set B	Set C	N/A		
	[George] a/b = 7/10 x/y = 1/10	[James] a/b = 4/8 x/y = 3/8	[Matt] a/b = 8/7 x/y = 5/7			
Interpretation	**Part-whole**	Ratio	Operator	Quotient	**Measure**	
Representation	All	Area	Number line	Sets	**Liquid measures**	Undefined
Task type	Procedural learning	Conceptual learning				
	Structured	Classify	Analyse	Interpret	Justify	**Construct**

Expected student behaviour:
Student will make a/b using liquid measures representation. They will make another representation using liquid measures that shows x/y. They will compare the two fractions to identify the solution. OR Student will know the difference and make the solution, x/y and a/b.

Example difficulties and opportunities:

- Student may think a/b is the solution and enter the fractions into the equivalence box inaccurately. Provide a prompt to ask the student to re-read the task.
- Student may expect Fractions Lab to provide the solution if they enter a/b and x/y as the subtrahend and difference. Feedback that Fractions Lab cannot provide the missing fraction.
- Student makes two fractions where the denominators, if one subtracted from the other as whole numbers, would equal the fraction required. E.g. 8/20 - 1/20 = 7/10: Ask student to check their answer, encouraging them to check the denominators.

Task completion:
The student will check their solution by using the subtraction box, entering [new fraction] - a/b = x/y.

Final Reflective prompt:	GENERIC	SPECIFIC
	Please explain how you would carry out this task using a different representation.	Is there another way you could have calculated the solution?

<table>
<tr><td>Task-specific vocabulary:</td><td colspan="2">• Subtract • Difference • Poured
• Take away</td></tr>
<tr><td>Misconceptions:</td><td>GLOBAL
• Treating numerator / denominator as whole numbers</td><td>SITUATED
• Subtracting across numerators and denominators</td></tr>
</table>

<table>
<tr><td>Task code: 3b+.1</td><td colspan="6">Coarse-grain goal: Add two fractions with denominators that are multiples of the same number</td></tr>
<tr><td colspan="7">6.1.12 Task description: [NAME] used [REP] to add a/b and x/y. Can you find out what her answer was?</td></tr>
<tr><td colspan="7">6.1.13 Task dimensions:</td></tr>
<tr><td>Fine-grain goal(s)</td><td>6. Expand fractions to find equivalents</td><td>12. Produce the sum of two fractions</td><td></td><td></td><td colspan="2"></td></tr>
<tr><td rowspan="2">Fraction Type</td><td>Set A</td><td>Set B</td><td>Set C</td><td colspan="3">N/A</td></tr>
<tr><td>a/b = 1/6
x/y = 5/12</td><td>a/b = 2/3
x/y = 2/9</td><td>a/b = 4/3
x/y = 3/6</td><td colspan="3"></td></tr>
<tr><td>Interpretation</td><td>Part-whole</td><td>Ratio</td><td>Operator</td><td>Quotient</td><td colspan="2">Measure</td></tr>
<tr><td>Representation</td><td>All</td><td>Area [April]</td><td>Number line [Clara]</td><td>Sets [June]</td><td>Liquid measures [Mary]</td><td>Undefined</td></tr>
<tr><td rowspan="2">Task type</td><td>Procedural learning</td><td colspan="5">Conceptual learning</td></tr>
<tr><td>Structured</td><td>Classify</td><td>Analyse</td><td>Interpret</td><td>Justify</td><td>Construct</td></tr>
<tr><td colspan="7">Expected student behaviour:
The student will make two fractions from the same representation: a/b and x/y. They will change a/b into an equivalent fraction so that it has the same denominator as x/y. They may make the equivalent fraction by copying and changing a/b or they may simply change partitioning,</td></tr>
<tr><td colspan="7">Example difficulties and opportunities:
• Student may add a/b and x/y without making an equivalent of a/b: encourage student to reflect on why Fractions Lab wouldn't add the two fractions together
• Student may try to reduce x/y into an equivalent fraction that shares a denominator with a/b: encourage student to reflect on why their new fraction is not equivalent to the original x/y by using the comparison box.</td></tr>
<tr><td colspan="7">Task completion:
The student will drag the two fractions (x/y and the fraction equivalent to a/b) into the addition box to check their answer is correct.</td></tr>
<tr><td rowspan="2">Final Reflective prompt:</td><td colspan="3">GENERIC</td><td colspan="3">SPECIFIC</td></tr>
<tr><td colspan="3">Please explain why you solved the problem this way.</td><td colspan="3">What changes did you have to make to your fractions? Why?</td></tr>
</table>

<table>
<tr><td>Task code: 3b-.1</td><td colspan="6">Coarse-grain goal: Subtract two fractions with denominators that are multiples of the same number</td></tr>
<tr><td colspan="7">6.1.14 Task description: Show a subtraction where the solution is a/b. The denominators should be multiples of the same number.</td></tr>
<tr><td colspan="7">6.1.15 Task dimensions:</td></tr>
<tr><td>Fine-grain goal(s)</td><td>6. Expand fractions to find equivalents</td><td>15. Produce the solution of subtracting two fractions</td><td></td><td></td><td></td><td></td></tr>
<tr><td rowspan="2">Fraction Type</td><td>Set A</td><td>Set B</td><td>Set C</td><td>N/A</td><td></td><td></td></tr>
<tr><td>[George]
a/b = 7/10
x/y = 1/10</td><td>[James]
a/b = 4/8
x/y = 3/8</td><td>[Matt]
a/b = 8/7
x/y = 5/7</td><td></td><td></td><td></td></tr>
<tr><td>Interpretation</td><td>Part-whole</td><td>Ratio</td><td>Operator</td><td>Quotient</td><td>Measure</td><td></td></tr>
<tr><td>Representation</td><td>All</td><td>Area</td><td>Number line</td><td>Sets</td><td>Liquid measures</td><td>Undefined</td></tr>
<tr><td rowspan="2">Task type</td><td>Procedural learning</td><td colspan="5">Conceptual learning</td></tr>
<tr><td>Structured</td><td>Classify</td><td>Analyse</td><td>Interpret</td><td>Justify</td><td>Construct</td></tr>
<tr><td colspan="7">Expected student behaviour:
The student will make two fractions out of the same representation of their own choice. The denominator of the two fractions will be different but share the same multiple. The difference of the two fractions will be a/b. They will need to change one of the fractions to ensure the denominators are the same before they use the subtraction box to check their working.</td></tr>
<tr><td colspan="7">Example difficulties and opportunities:
• Student may make two fractions with the same denominator: encourage them to re-read the task and highlight that the two denominators should not be the same. Provide example if necessary.
• Student may add two fractions to make a/b. Reinforce that the task is to subtract.
• Student may use preferred representation. Encourage to try using a different representation.</td></tr>
<tr><td colspan="7">Task completion:
The student will check their solution by using the subtraction box, entering their two fractions and the solution a/b.</td></tr>
<tr><td rowspan="2">Final Reflective prompt:</td><td colspan="3">GENERIC</td><td colspan="3">SPECIFIC</td></tr>
<tr><td colspan="3">What have you learnt about taking away fractions?</td><td colspan="3">How did you decide on the fractions you used?</td></tr>
<tr><td>Task-specific vocabulary:</td><td colspan="6">• Subtract • Difference • Poured
• Take away</td></tr>
</table>

Task code: 3b-.1	**Coarse-grain goal: Subtract two fractions with denominators that are multiples of the same number**	
6.1.14 Task description: Show a subtraction where the solution is a/b. The denominators should be multiples of the same number.		
6.1.15 Task dimensions:		
Misconceptions:	GLOBAL • Treating numerator / denominator as whole numbers	SITUATED • Subtracting across numerators and denominators

Task code: 3c+.1	Coarse-grain goal: Add two fractions with unlike denominators					
6.1.16 Task description: [NAME] used [REP] to think about adding fractions. Her answer is shown here. One of her fractions was a/b. Can you make it too?						
6.1.17 Task dimensions:						
Fine-grain goal(s)	6. Expand fractions to find equivalents	10. Cancel down to find equivalents	12. Produce the sum of two fractions			
Fraction Type	Set A	Set B	Set C	N/A		
	a/b = 1/4 (for answer 7/12)	a/b = 1/3 (for answer 7/12)	a/b = 7/12 (for answer 4/3)			
Interpretation	**Part-whole**	Ratio	Operator	Quotient	Measure	
Representation	**All**	Area [Isra]	Number line [Aster]	Sets [Jess]	Liquid measures [Sara]	Undefined
Task type	Procedural learning	Conceptual learning				
	Structured	Classify	Analyse	Interpret	Justify	**Construct**
Expected student behaviour: The student will make fraction a/b. They may make the equivalent fraction by copying and changing a/b or they may simply change by partitioning. The equivalent fraction will have a denominator equal to b. They make a second fraction that is the difference between the answer and a/b, using a fraction with the same denominator. They simplify (cancel down) the fraction.						
Example difficulties and opportunities: • Student uses preferred representation: Encourage student to use a different representation. • Student adds the given fraction with a/b: Encourage to think about what the task is asking them to do. • Student may not simplify the new fraction: Explain the student used the simplest fraction.						
Task completion: The student will drag the two fractions (a/b and the new simplified fraction they have made) and the solution fraction given into the addition box to check their answer is correct.						
Final Reflective prompt:	GENERIC	SPECIFIC				
	What have you learnt about adding fractions when the denominators are different?	You have expanded and simplified fractions in this task. What have you learnt doing this?				

Task code: 3c-.1	Coarse-grain goal: Subtract two fractions with the same denominator					
6.1.18 Task description: [NAME] is using [REP] to think about subtracting fractions. He wants to work out a/b - c/d. Can you show him?						
6.1.19 Task dimensions:						
Fine-grain goal(s)	6. Expand fractions to find equivalents	10. Cancel down to find equivalents	15. Produce the solution of subtracting two fractions			
Fraction Type	Set A a/b = 2/3 c/d = 1/2	Set B a/b = 6/8 c/d = 1/5	Set C a/b = 10/4 c/d = 6/5	N/A		
Interpretation	**Part-whole**	Ratio	Operator	Quotient	**Measure**	
Representation	**All**	Area [Zach]	Number line [Rohan]	Sets [Elliott]	Liquid measures [Jack]	Undefined
Task type	Procedural learning	Conceptual learning				
	Structured	Classify	Analyse	Interpret	Justify	**Construct**

Expected student behaviour:
Student will make a/b and c/d. They will make equivalent fractions of a/b and c/d so the new fractions share the same denominator. They will make a third fraction (x/y, the solution) that shares the same denominator as the other two. This fraction should be the difference between a/b and c/d.

Example difficulties and opportunities:

- Student may think a/b is the solution and enter the fractions into the equivalence box inaccurately. Provide a prompt to ask the student to re-read the task.
- Student may expect Fractions Lab to provide the solution if they enter a/b and c/d as the subtrahend and difference. Feedback that Fractions Lab cannot provide the missing fraction.
- Student may treat numerators/denominators as whole numbers and subtract across, e.g 2/3 – ½ = 1/1. Recommend to the student they check their solution using the subtraction box.

Task completion:
The student will check their solution by using the subtraction box, entering [new fraction] - a/b = c/d.

Final Reflective prompt:	GENERIC	SPECIFIC
	What would you explain to [NAME] about subtracting fractions where the denominators are different?	Why did your first idea not find the solution you needed?

Task-specific vocabulary:	• Subtract • Difference • Take away	
Misconceptions:	GLOBAL • Treating numerator / denominator as whole numbers	SITUATED • Subtracting across numerators and denominators