6 Number: fractions, decimals and percentages

In the Early Years Foundation Stage (DfE, 2012, page 6) practitioners are expected to provide children with opportunities to develop and improve their skills in counting, understanding and using numbers, calculating simple addition and subtraction problems, and describing shapes, spaces and measures. Problems with simple fractions are often met in these contexts. In Key Stage 1 (Years 1 and 2) children are introduced to early notions of fractions through finding simple fractions of objects, numbers and quantities. During lower Key Stage 2 (Years 3 and 4) children develop their ability to solve a range of problems, including some with simple fractions and decimal place value. Children will develop the connections between multiplication and division with fractions, decimals, percentages and ratio in upper Key Stage 2 (Years 5 and 6). By the end of Year 6, children should be fluent in working with fractions, decimals and percentages.

Fractions

It is important to remember that most children will have met fractions informally in an everyday context before coming to school and teachers need to be aware of the potential for misconceptions arising from these encounters. For example, a parent may encourage a child to choose the 'biggest half' of a food item they are sharing. However, this is incorrect as mathematically, the halves should be equal.

Lamon (2001) argues that *traditional instruction in fractions does not encourage meaningful performance* (page 146). She researched the effect of teaching fractions for understanding and found that where learning was underpinned by understanding, children were able to solve problems involving more complex fractions. Similarly, Critchley (2002) describes how a *real* problem-solving activity involving the use of fractions supported a child's ability to understand and use them.

There are many different interpretations of fractions and it is important that teachers are both aware of and understand these interpretations so they can introduce them to children in a meaningful way. Teachers also need to be aware that some interpretations of fractions are conceptually more difficult than others. The relative conceptual difficulty of particular interpretations of fractions is widely recognised by mathematics educators (Charalambous and Pitta-Pantazi, 2005). The interpretations (Lamon, 1999) are:

Part of a whole: here an object is 'split' into two or more equal parts.

Part of a set of objects: what part of the set of objects has a particular characteristic?

Number on a number line: numbers which are represented *between* whole numbers.

Operator: the result of a division.

Ratio: comparing the relative size of two objects or sets of objects.

The first interpretation in this list is that a fraction is part of a whole. Charalambous and Pitta-Pantazi's (2005) research identified that while this way of teaching fractions was necessary, it was not appropriate to use this as the only way in to teaching the other interpretations of fractions. They explained that it was necessary for teachers *to scaffold students to develop a profound understanding of the different interpretations of fractions, since such an understanding could also offer to uplift students' performance in tasks related to the operations of fractions* (Charalambous and Pitta-Pantazi, 2005, page 239).

Fractions are among the most difficult mathematical concepts that children come across at primary school (Charalambous and Pitta-Pantazi, 2005). In fact, analysis of children's errors in fractions has been investigated for many years (see e.g. Brueckner, 1928; Morton, 1924). Nickson (2000) suggests that children have difficulty applying their knowledge of fractions to problem-solving situations because there are several interpretations of fractions and therefore children do not know which interpretation to use. Lamon (2001, pages 147–148) explains that even students who are studying for a degree in mathematics may have a limited understanding of fractions.

Decimals

Decimal numbers are an extension of the whole-number place-value system. Bailey and Borwein (2010) argue that the modern system of decimal notation with zero, together with basic computational schemes, was the greatest discovery in mathematics. This happened over 1500 years ago in India, around 500 BC.

Decimal numbers are symbolic representations of units less than one (rational numbers) in the same way that the whole-number place-value system represents quantities of objects. The conceptual ideas underpinning decimals are the same as those underpinning fractions, for example part of a whole, part of a set, and so on (see the discussion in the 'Fraction' section above). In effect, decimals are simply another way of representing fractions in written form. The implication of this is that children need to have a sound understanding of fractions in order to use abstract decimal notation to represent fractions. Pagni (2004) argues that fractions and decimals should not be taught separately. Since fractions and decimals are representations for the same numbers, Pagni suggests teachers should show the connection between them by placing equivalent fractions and decimals on a number line.

Errors in the use of decimals are likely to have two sources of misunderstanding: place value and fractions (Moloney and Stacey, 1997). Where such errors occur, teachers might consider returning to much earlier concepts to ensure the child has sufficient understanding of these ideas to be able to use decimal notation.

According to Sadi (2007), more children have difficulties with decimals than any other number concept. He suggests that this is because of the gap in understanding between the natural numbers and decimal numbers.

Percentages

The term 'per cent' means literally 'for every hundred'. Percentages are conceptually equivalent to the part-of-a-set interpretation of fractions – if our set contains 100 objects then we can easily see that if 50 of the objects have a particular attribute then 50 per cent (%) of the set can be said to have the attribute. Equally, we can relate percentages to the part–whole representation of fractions in that 'the whole' represents 100%. Percentages are used widely in 'everyday' life and children will have an understanding that 30% off in the sale means that the item is reduced in price. This does not mean that children understand the mathematics of percentages, just as a young child who can 'reel off' the numbers to 20 may not be able to count. Like decimals, percentages are a method of representing rational numbers, so children need to have a sound understanding of fractions before being formally introduced to percentages.

In primary school, the focus should be on helping children to understand the notion of per cent and relating this to simple fractional amounts, for example halves, quarters and tenths. It is tempting to think that we might introduce children to a mathematical formula in order to solve more complex percentage problems, but this should be avoided – it might be argued that teaching in this way leads to weak understanding and negative attitudes towards percentages. Instead, just as reported earlier in the 'Decimals' section, it is useful to introduce equivalent fractions, decimals and percentages together. Van den Heuvel-Panhuizen (2003) explores a Dutch Realistic Mathematics Education (RME) learning trajectory in which Grade 5 children were taught percentages through a process drawing on the children's own experiences of using percentages, decimals and fractions, reporting that it was *remarkable how easily the children got to work so easily … everything happened very naturally, and it was clear from the way in which the children discussed {the assignment}… that they knew what the percentages represented* (page 20).

White *et al*. (2007) discuss how the multiplicative relationship of percentages causes children difficulties. They cite Misailidou and Williams (2003), who showed how 10–14-year-olds often used inappropriate additive strategies.

Another difficulty is with the terms '**of**' and '**out of**' – both of which represent an operator which needs explaining:

> '**of**' represents the multiplication operator, for example 50% **of** 80 means 0.5 × 80;
>
> '**out of**' represents the division operator, for example 40 **out of** 80 means 40 ÷ 80.

A key aspect of teaching percentages in the primary school is to help children to understand the relationship between fractions and percentages. Children need to understand that 50% is equivalent to one-half and 25% is equivalent to one-quarter. This enables children to solve simple percentage problems using a conceptual model based on their understanding of fractions, thus avoiding 'the formula'. If teachers wish to explore more complex problems with some children, they can then use the conceptual model to derive 'the formula' for 'of' problems, so that becomes a natural progression in understanding. Some children will find the concept of percentages very difficult. This may be because their understanding of fractions is weak. In these cases, teachers will need to return to earlier concepts relating to fractions, including concrete representations of part of a set and part of a whole.

Ratio and proportion

Ratio and proportion are no longer an explicitly taught aspect in the new national curriculum for England. However, in the non-statutory notes and guidances, ratio and proportion are contextualised in other problems, such as scale drawings, similar shapes, recipes and comparing quantities.

Lamon (2008) explores how the key to understanding fractions, decimals, percentages, ratio and proportion is discussion during reasoning activities. Her book, developed after years of research on this area, develops teacher confidence by asking the same sorts of questions that teachers may ask children. She explains how ratio is a comparison of any two quantities and that it expresses an idea that cannot be expressed by a single number. Additionally, a key premise of the book is about using proportion as a powerful mathematical tool. For example, to know that when two quantities are related to each other and one changes, the other also changes in a precise way, and knowing that the relationship does not change but that the quantities may increase or decrease, this aids the development of proportional reasoning.

You may find these practical examples helpful.

Ratio: 'for every' – there are three pencils for every child in the class.

Proportion: 'in every' – there are two red pencils in every pot of pencils.

Much research in the area of proportional reasoning identifies that solving ratio and proportion problems is a very difficult task for most children and student teachers (Misailidou and Williams, 2002). Similarly to the research discussed above on percentages, Ryan and Williams (2007) also identify multiplicative versus additive strategies as a barrier to solving ratio and proportion problems. They explain how, on occasion, additive strategies can work (for example in the case of 1:2 or 1:*n*, it is possible to count on in order to solve the problems) but that most problems in any ratio *m:n* cannot work by using additive strategies. Their research identifies how many children prefer to add or subtract numbers in problems, and avoid multiplication, division and particularly fractions.

6.1 Fractions as part of a shape

The teacher asks the children to divide a semicircle into quarters. One child's response is given in Figure 6.1.

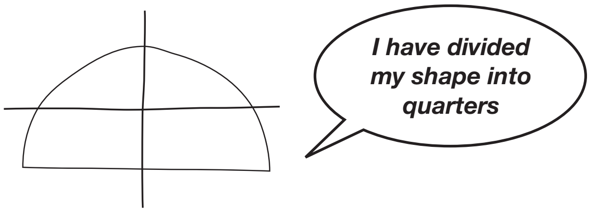


Figure 6.1

The error

The child has not divided the semicircle into four equal parts.

Why this happens

The child does not understand that all the parts must be equal. He may not have had sufficient experience of dividing physical objects into equal parts where the parts can be directly compared to each other. Additionally, he may be used to dividing squares, oblongs and circles so he thinks the methods he has used for these shapes works for all shapes. In effect, he incorrectly generalises the methods he has used for squares, oblongs and circles.

Curriculum links

EYFS	They solve problems, including doubling, halving and sharing
Year 1	Recognise, find and name a half as one of two equal parts of an object, shape or quantity
Year 1	Recognise, find and name a quarter as one of four equal parts of an object, shape or quantity
Year 2	Recognise, find, name and write fractions $^1/_3$, $^1/_4$, $^2/_4$ and $^3/_4$ of a length, shape, set of objects or quantity

6.2 Fractions as part of a set of objects

The teacher asks two children what fraction of the set in Figure 6.2 is black. They say one-third.

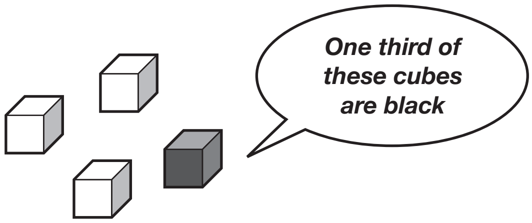


Figure 6.2

The error

The children have failed to take the complete set of objects as the whole unit.

Why this happens

The children have compared the one black cube against the three white cubes and concluded that one-third of the set of objects is black. This misconception may be common in children up to about seven years of age and may be related to Piaget's findings from his class-inclusion task. Piaget found that if children are presented with a set of say five red objects and two blue objects and asked if there are more red objects or more 'objects', children will say there are more red objects. This is because they compare the red objects with the blue objects instead of comparing the red objects with the total number of objects in the set.

Curriculum links

EYFS	They solve problems, including doubling, halving and sharing
Year 2	Recognise, find, name and write fractions 1/3, 1/4, 2/4 and 3/4 of a length, shape, set of objects or quantity
Year 3	Recognise, find and write fractions of a discrete set of objects: unit fractions and non-unit fractions with small denominators

6.3 Fractions as numbers on a number line

The teacher is using a counting stick in the oral/mental part of the daily mathematics lesson (Figure 6.3). She tells children that one end of the counting stick represents 0 and the other end of the counting stick represents 10. She asks one child to place '½' on the counting stick. He places his card in the centre of the counting stick.

The error

The child has placed his card halfway along the counting stick instead of halfway between 0 and 1.

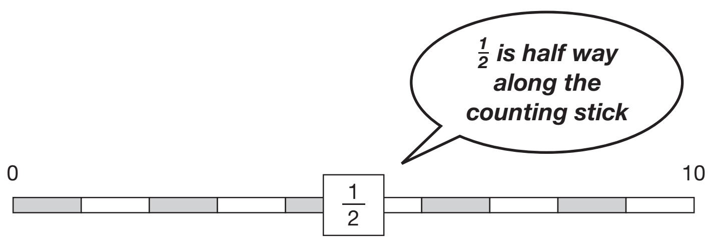


Figure 6.3

Why this happens

When children are introduced to fractions they are introduced to unit fractions such as one-half or one-quarter. They sometimes believe that a fraction is a number smaller than one, i.e. between 0 and 1. Children can usually successfully identify fractions on a number line between 0 and 1. The difficulty arises when the number line is extended to include numbers greater than 1. It is likely that the child's previous experience has involved halving shapes, and when a shape had been halved, there has been a line drawn in the middle. The child has applied this knowledge to the number line, instead of treating ½ as a number in its own right with a specific place on the number line half way between 0 and 1.

Curriculum links

Year 3	Recognise and use fractions as numbers: unit fractions and non-unit fractions with small denominators

6.4 Fractions as ratios

The teacher asks the class to compare two sets of objects, A and B (Figure 6.4).

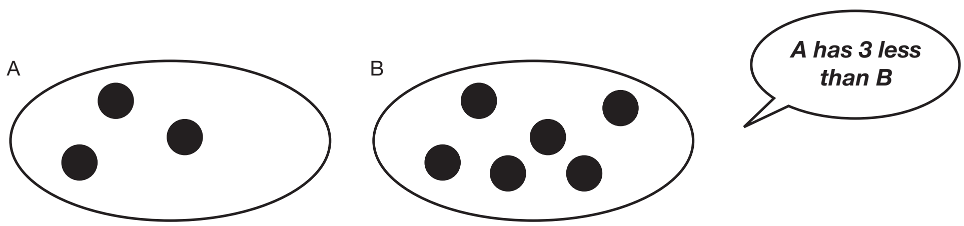


Figure 6.4

One child tells the teacher that set A is 3 less than set B.

The error

The child has used additive comparison instead of multiplicative comparison.

Why this happens

The correct response should be that set A has half as many dots as set B. The child is drawing on her knowledge of addition and subtraction from earlier in her education – when comparing the size of two sets of objects involved using addition/subtraction. She may do this because she does not fully understand the nature of the task. This type of error may also arise when we are comparing lengths. For example, a child may say that object A is 12cm longer than object B instead of saying that object A is 5 times as long as object B. See also error 5.1.

Curriculum links

Year 6	Solve problems involving unequal sharing and grouping using knowledge of fractions and multiples

6.5 Reading fractions (1)

Two children are working with their teacher shading in parts of shapes (see Figure 6.5). The teacher asks them what fraction of the shape has been shaded. One of them tells the teacher that one-'twoth' of the shape has been shaded.

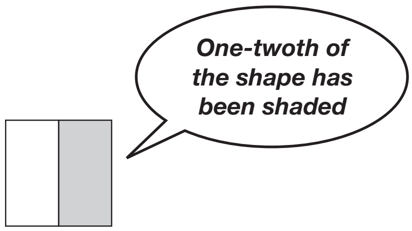


Figure 6.5

The error

The child is trying to apply a consistent naming system to a system which is not fully consistent.

Why this happens

The child is actually thinking logically. Furani (2003) explores how naming and misnaming involve logic and rules, and are often an aid to supporting children's mathematical learning. Unfortunately, there are inconsistencies in the English conventions of naming fractions and this can confuse children. Indeed, in American English, one-quarter is referred to as 'one fourth'. The child needs to learn that we use the term 'half' to represent 1 out of 2. Another common error with naming fractions is the use of 'one whole'. Sometimes children interpret this as 'one hole'.

Curriculum links

Year 2	Write simple fractions, for example: $^1/_2$ of 6 = 3 and recognise the equivalence of $^2/_4$ and $^1/_2$

6.6 Reading fractions (2)

The child writes $^2/_{10}$ and says 'two tens'.

Figure 6.6

The error

The child uses whole numbers, rather than the fraction's name, when reading it.

Why this happens

The child may simply not have the vocabulary of 'tenth' to read the fraction accurately. It is likely that she sees the numbers in the fraction as two unrelated whole numbers separated by a line, rather than as a (fractional) number in its own right.

Curriculum links

Year 2	Write simple fractions for example, $^1/_2$ of 6 = 3 and recognise the equivalence of $^2/_4$ and $^1/_2$

6.7 Reading fractions (3)

The children are working together as a class and the teacher is asking them to read the fractions he displays. One child says 2/3 is 'two and three' and another says 'two slash three'.

The error

The children are using whole numbers, rather than the fraction's name, when reading it.

Why this happens

The child may simply not have the vocabulary of 'third' to read the fraction accurately. It is likely that they see the numbers in the fraction as two unrelated whole numbers separated by a line, rather than as a (fractional) number in its own right.

Curriculum links

Year 2	Write simple fractions for example, $^1/_2$ of 6 = 3 and recognise the equivalence of $^2/_4$ and $^1/_2$

6.8 Representing fractions

The children are drawing fractions using shapes (see Figure 6.7). The teacher asks one of them what fraction she has drawn and she tells the teacher that it shows one-third.

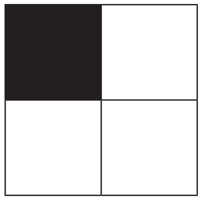


Figure 6.7

The error

The child sees one part shaded and three parts not shaded as the fraction 1/3.

Why this happens

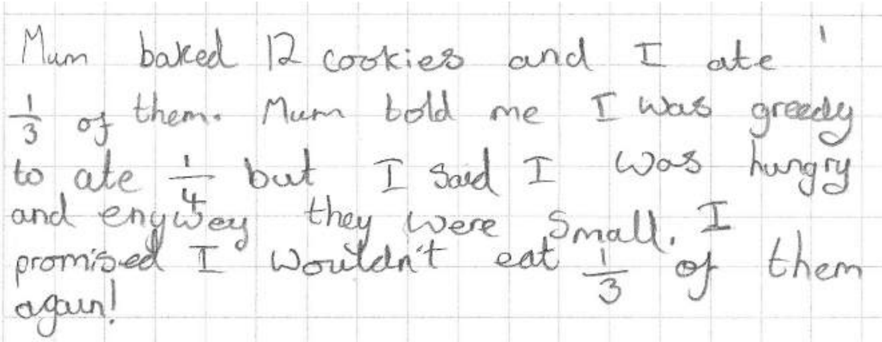
There are several reasons why the child has made this error. The most likely reason is that the child doesn't see the four parts as all part of the whole. Understanding the whole and that fractions are a part of the whole is very important, but here, the child may have focused on the individual parts of the square. Another reason is that the child may have only just begun to focus on thirds at school. In this case she may be overgeneralising her knowledge of fractions (as quarters and halves only) to draw the square, divide it, and try to make an unfamiliar fraction in some logical manner. Finally, the child may be thinking of 1/3 as a ratio – 1:3 – and therefore showing a ratio of one to three which, in the fraction context, is incorrect.

Curriculum links

Year 1	Recognise, find and name a half as one of two equal parts of an object, shape or quantity
Year 2	Recognise, find and name a quarter as one of four equal parts of an object, shape or quantity
Year 3	Recognise, find, name and write fractions $^1/_3$, $^1/_4$, $^2/_4$ and $^3/_4$ of a length, shape, set of objects or quantity

6.9 Writing and using fractions

When asked to write a story using the fraction *one-third*, this child produced a story using *one-third* and *one-quarter* interchangeably.



Mum baked 12 cookies and I ate $\frac{1}{3}$ of them. Mum told me I was greedy to ate $\frac{1}{4}$ but I said I was hungry and enyway they were Small. I promised I wouldn't eat $\frac{1}{3}$ of them agun!

Figure 6.8

The error

The child has used *one-third* and *one-quarter* interchangeably in their story.

Why this happens

The child may have done this because the words 'quarter' and 'third' do not suggest the numbers 4 and 3 and therefore they have not realised that the two fractions (that they have heard used in other contexts) have different meanings.

Curriculum links

Year 3	Recognise, find, name and write fractions $\frac{1}{3}$, $\frac{1}{4}$, $\frac{2}{4}$ and $\frac{3}{4}$ of a length, shape, set of objects or quantity

6.10 Ordering fractions

The children have been given some cards with unit fractions written on them (Figure 6.9). The teacher asks the children to look at the two cards and say which is the biggest.

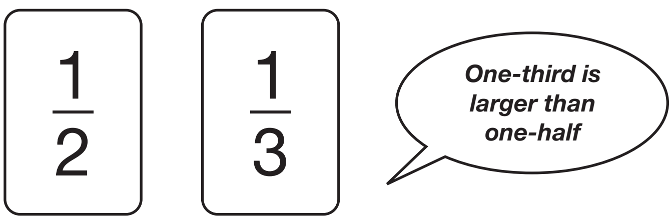


Figure 6.9

The error

The child believes that one-third is bigger than one-half because the denominator of one-third is larger than the denominator of one-half.

Why this happens

The child is using his knowledge of whole numbers to order the fractions. He is over- generalising whole-number concepts. The child does not understand the written notation of fractions. He needs to know what the notation means – that we have one part **out of** three equal parts. The child probably does not have a clear understanding of the concept of fractions in a practical sense.

Associated with this idea, children also need to realise that 'the line' in a fraction represents division. So ¾ means 3 **out of** 4, which is also 3 **divided by** 4.

Curriculum links

Year 3	Compare and order unit fractions, and fractions with the same denominators
Year 6	Compare and order fractions whose denominators are all multiples of the same number

6.11 Comparing fractions

The children have been asked to compare the fractions and write a sentence about them (Figure 6.10A and Figure 6.10B).

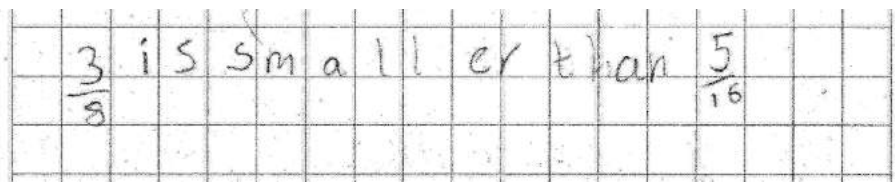


Figure 6.10A

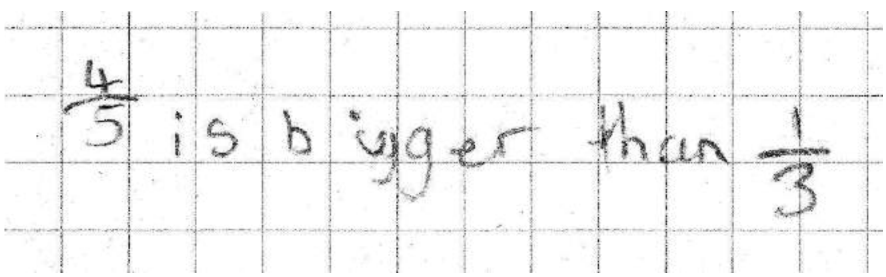


Figure 6.10B

The errors

Child A states that $^3/_8$ is smaller than $^5/_{16}$. Child B states that $^4/_5$ is bigger than $^1/_3$.

Why this happens

It is likely that Child A is focusing only on the numerator and dismissing the denominator, to show that 3 is smaller than 5, and that Child B is focusing only on the denominator to show that 4 is greater than 1. Both children are showing a lack of understanding of fractions, treating the two parts of each fraction as a whole number. The children need to know what the notation means – that we have *x* parts (the numerator) **out of** *y* equal parts (the denominator).

Associated with this idea, children also need to realise that ‘the line’ in a fraction represents division. So ¾ means 3 **out of** 4, which is also 3 **divided by** 4.

Curriculum links

Year 3	Compare and order unit fractions, and fractions with the same denominators
Year 6	Compare and order fractions whose denominators are all multiples of the same number

6.12 Converting mixed numbers to proper fractions

A child writes:

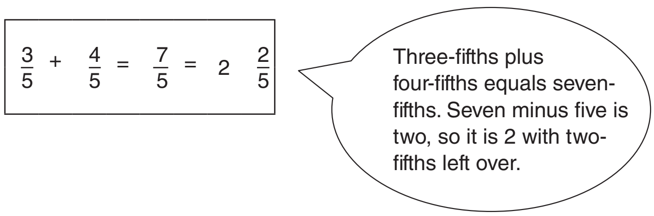


Figure 6.11

The error

The child states that $^7/_5$ is equivalent to 2 $^2/_5$.

Why this happens

Here the child has used subtraction instead of division to work out the mixed number. This can happen when children think of the numerator and denominator as whole numbers, rather than parts of a fraction.

Curriculum links

Year 5	Recognise mixed numbers and improper fractions and convert from one form to the other and write mathematical statements > 1 as a mixed number [for example, $^2/_5 + {}^4/_5 = {}^6/_5 = 1\ {}^1/_5$]
Year 6	Add and subtract fractions with different denominators and mixed numbers, using the concept of equivalent fractions

6.13 Equivalent fractions (1)

While using a computer program called 'Fractions Lab' (see link.lkl.ac.uk/FractionsLab) a child said, '$^2/_6 = {}^1/_{12}$. The computer is wrong because $2 \times 6 = 1 \times 12$'.

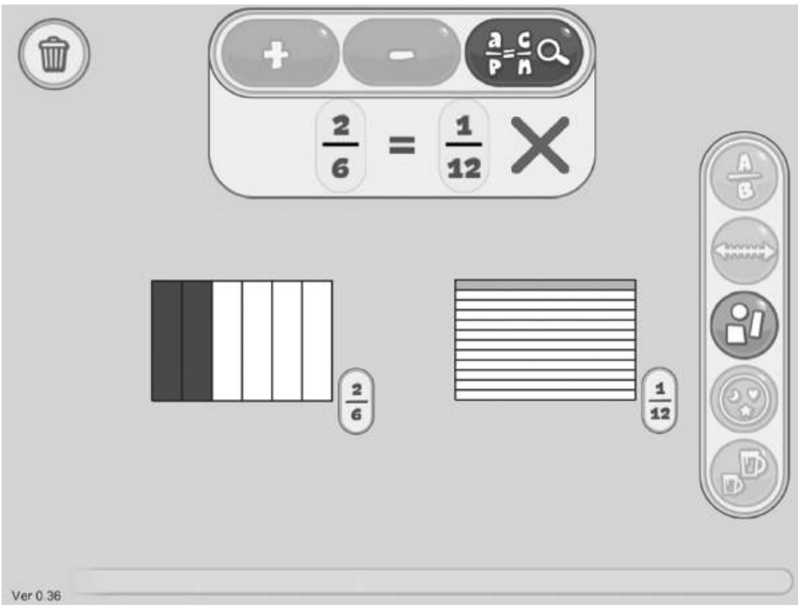


Figure 6.12

The error

The child states the computer is wrong because $2 \times 6 = 1 \times 12$.

Why this happens

Here the child is treating the numerator and denominator as if they were whole numbers. He knows that 2 multiplied by 6 is equal to 1 multiplied by 12, and is treating the vinculum (the fraction bar) as a multiplier.

Curriculum links

Year 2	Recognise, find, name and write fractions $^1/_3$, $^1/_4$, $^2/_4$ and $^3/_4$ of a length, shape, set of objects or quantity
Year 3	Recognise and show, using diagrams, equivalent fractions with small denominators
Year 4	Recognise and show, using diagrams, families of common equivalent fractions
Year 5	Identify, name and write equivalent fractions of a given fraction, represented visually, including tenths and hundredths
Year 6	Use common factors to simplify fractions; use common multiples to express fractions in the same denomination

6.14 Equivalent fractions (2)

A child explains, '$^6/_7 = {}^8/_9$ because 7 take 6 is one and 9 take 8 is one'.

The error

The child states that $^6/_7$ is equivalent to $^8/_9$ because the two parts have a difference of 1.

Why this happens

Here the child is treating the numerator and denominator as if they were whole numbers. She knows that the difference between 6 and 7 is 1, and the difference between 8 and 9 is also 1. Therefore, she has concluded that they are equivalent.

Curriculum links

Year 2	Recognise, find, name and write fractions $^1/_3$, $^1/_4$, $^2/_4$ and $^3/_4$ of a length, shape, set of objects or quantity
Year 3	Recognise and show, using diagrams, equivalent fractions with small denominators
Year 5	Identify, name and write equivalent fractions of a given fraction, represented visually, including tenths and hundredths

6.15 Equivalent fractions (3)

A child does not believe that 'Fractions Lab' (link.lkl.ac.uk/FractionsLab) is right when it shows that $^6/_8$ and $^9/_{12}$ are equal.

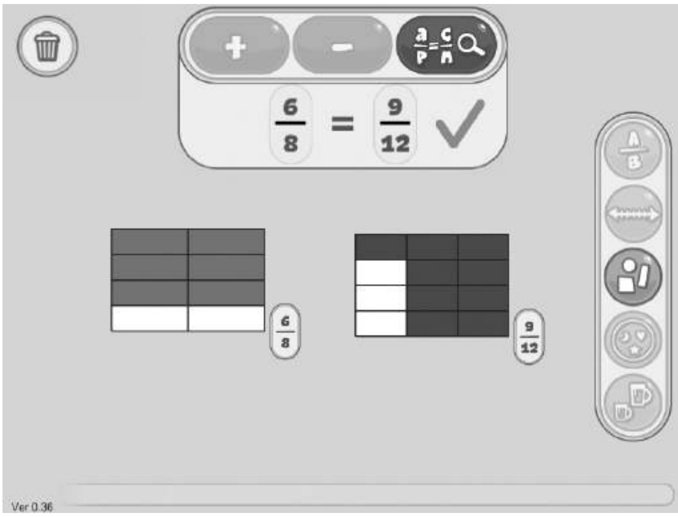


Figure 6.13

The error

The child says, '6 + 6 = 12, but 8 + 8 ≠ 9'.

Why this happens

Here the child is treating the numerator and denominator as if they were whole numbers and using addition to try and explain the relationship between the two fractions, which is unhelpful. This is common, and children should be encouraged to think about using multiplicative structures to explain relationships between fractions rather than additive structures.

Curriculum links

Year 2	Recognise, find, name and write fractions $^1/_3$, $^1/_4$, $^2/_4$ and $^3/_4$ of a length, shape, set of objects or quantity
Year 3	Recognise and show, using diagrams, equivalent fractions with small denominators
Year 4	Recognise and show, using diagrams, families of common equivalent fractions
Year 5	Identify, name and write equivalent fractions of a given fraction, represented visually, including tenths and hundredths
Year 6	Use common factors to simplify fractions; use common multiples to express fractions in the same denomination

6.16 Converting fractions to decimals (1)

The children have been asked to complete a series of conversions of fractions to decimals.

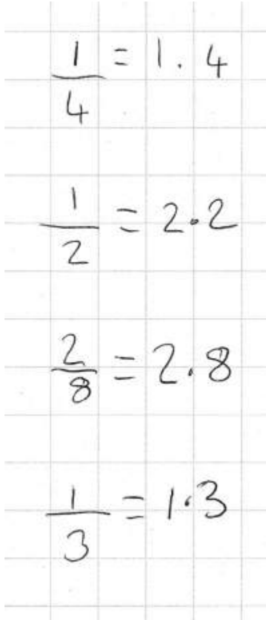


Figure 6.14

The error

The child has replaced the vinculum (the fraction bar) with a decimal point.

Why this happens

The child does not appear to understand that the conventions for writing fractions and decimals are not interchangeable. The child appears to have focused on the numbers in the fractions as whole numbers and has not demonstrated an understanding of fractions or decimal numbers as part of a whole.

Curriculum links

Year 4	Recognise and write decimal equivalents to $^1/_4$, $^1/_2$ and $^3/_4$
Year 5	Read and write decimal numbers as fractions [for example, 0.71 = $^{71}/_{100}$]
Year 6	Associate a fraction with division and calculate decimal fraction equivalents [for example, 0.375] for a simple fraction [for example, $^3/_8$]

6.17 Converting fractions to decimals (2)

A child is asked to find the decimal equivalent of $^7/_2$. The child responds, 'Seven halves are 3.1 because there are three wholes and one left over' and draws the representation shown in Figure 6.15.

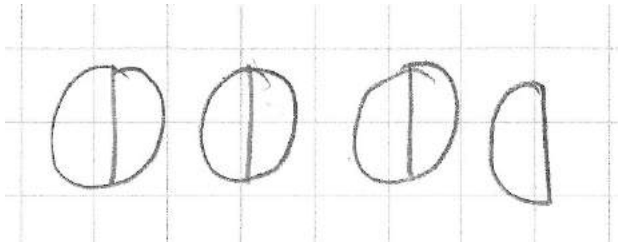


Figure 6.15

The error

The child has correctly identified that there are three wholes, but has misinterpreted the 'one left over' as 0.1.

Why this happens

The child has not used 0.5 for ½. This may be because using remainders in whole number division is more familiar to the child and they have applied that knowledge in this situation. The child may not know that ½ = 0.5 (or that 0.1 = $^1/_{10}$) in this context, and settles on what they deem to be the most sensible solution: that 0.1 represents one left over.

Curriculum links

Year 4	Solve problems involving increasingly harder fractions to calculate quantities, and fractions to divide quantities, including non-unit fractions where the answer is a whole number
Year 5	Read and write decimal numbers as fractions [for example, 0.71 = $^{71}/_{100}$]
Year 6	Associate a fraction with division and calculate decimal fraction equivalents [for example, 0.375] for a simple fraction [for example, $^3/_8$]

6.18 Converting fractions to decimals (3)

The teacher is working with a focus group and asked them to write in decimal form: 756 + $^4/_{100}$.

The children discuss various answers, including:

(a) 756.004

(b) 756.25

The errors

The children have

(a) incorrectly calculated the 4/100 is 0.004

(b) calculated that 100 divided by 4 gives 1/25 or 0.25

Why this happens

In (a) the children may have focused on the number of zeros in 100, rather than on the effect of dividing by 100. In (b) the children may have simplified the fraction and then concluded that 0.25 = $^{1}/_{25}$. Alternatively, the children are most familiar with division problems where the dividend is larger than the divisor, and the quotient (solution) is smaller. Therefore, they have jumped to the conclusion that they should be answering 100 ÷ 4 instead of 4 ÷ 100 and placed 0.25 at the end of the number.

Curriculum links

Year 3	Count up and down in tenths; recognise that tenths arise from dividing an object into 10 equal parts and in dividing one-digit numbers or quantities by 10
Year 4	Count up and down in hundredths; recognise that hundredths arise when dividing an object by one hundred and dividing tenths by ten.
Year 4	Recognise and write decimal equivalents of any number of tenths or hundredths
Year 4	Find the effect of dividing a one- or two-digit number by 10 and 100, identifying the value of the digits in the answer as ones, tenths and hundredths
Year 5	Read and write decimal numbers as fractions [for example, 0.71 = $^{71}/_{100}$]
Year 6	Associate a fraction with division and calculate decimal fraction equivalents [for example, 0.375] for a simple fraction [for example, $^{3}/_{8}$]

6.19 Converting fractions to decimals (4)

The teacher notices that a number of children are writing incorrect answers on their whiteboards during a whole-class interactive teaching session (Figure 6.16).

$$\frac{23}{100} = 0.0023$$
$$\frac{64}{100} = 0.0064$$
$$\frac{7}{100} = 0.007$$

Figure 6.16

The error

The children have provided answers that relate to thousandths, not hundredths.

Why this happens

The children may have focused on the number of zeros in 100 and inserted them after the decimal point. The children may have thought that the third digit to the left of the decimal point is in the hundreds column, so the third column to the right must be the hundredths. They may have also been working with converting thousandths and confused them with hundreths.

Curriculum links

Year 3	Count up and down in tenths; recognise that tenths arise from dividing an object into 10 equal parts and in dividing one-digit numbers or quantities by 10
Year 4	Count up and down in hundredths; recognise that hundredths arise when dividing an object by one hundred and dividing tenths by ten.
Year 4	Recognise and write decimal equivalents of any number of tenths or hundredths
Year 4	Find the effect of dividing a one- or two-digit number by 10 and 100, identifying the value of the digits in the answer as ones, tenths and hundredths
Year 5	Read and write decimal numbers as fractions [for example, 0.71 = $^{71}/_{100}$]
Year 5	Recognise and use thousandths and relate them to tenths, hundredths and decimal equivalents
Year 6	Identify the value of each digit in numbers given to three decimal places and multiply and divide numbers by 10, 100 and 1000, giving answers up to three decimal places

6.20 Adding fractions using a common denominator

The following examples illustrate only a selection of errors seen that are related to children adding fractions where a common denominator should be found.

a) $½ + {}^2/_3 = {}^2/_5$; $½ + {}^1/_4 = {}^1/_6$
b) $½ + {}^2/_3 = {}^3/_6$
c) $½ + {}^2/_3 = {}^2/_6$
d) $½ + {}^2/_3 = {}^3/_5$; $½ + {}^1/_4 = {}^2/_6$
e) $½ + {}^2/_3 = 1 + 2$
f) $½ + {}^2/_3 = {}^{(1 + 2)}/_8$, $½ + {}^1/_4 = {}^{(1 + 1)}/_8$
g) $½ + {}^1/_4 = {}^8/_4$

Figure 6.17

The errors

(a) The numerators are multiplied and the denominators are added.

(b) The numerators are added and the denominators are multiplied.

(c) The numerators and the denominators are multiplied, respectively.

(d) The numerators and the denominators of the given fractions are added, respectively.

(e) The numerators are added and the denominators are ignored.

(f) A common denominator is obtained by adding all denominators and numerators; the numerators remain untouched and are added to each other at the end.

(g) The common denominator is obtained correctly; the new numerators are obtained by adding the numerator and denominator in each fraction, respectively, i.e. $^1/_2 + {}^1/_4 = {}^{(3+5)}/_4$.

Why this happens

In these examples, the children have not used a common denominator accurately to find the solution. This may be due to the child having learnt the algorithm but forgetting to use it in these instances. It might also be that they do not realise the need to find a common denominator to make the sum easier to calculate. For (c), the child may have confused the multiplication algorithm. For (d) the child has attempted to find a common denominator, by adding all the numerators and denominators. Finally, the children all lack understanding of fractions and what the numerator and denominator represent.

Curriculum links

Year 3	Add and subtract fractions with the same denominator within one whole [for example, $^5/_7 + {}^1/_7 = {}^6/_7$]
Year 4	Add and subtract fractions with the same denominator
Year 5	Recognise mixed numbers and improper fractions and convert from one form to the other and write mathematical statements > 1 as a mixed number [for example, $^2/_5 + {}^4/_5 = {}^6/_5 = 1\,{}^1/_5$]

6.21 Adding fractions with unlike denominators

The child has added ¼ + ½ and written the answer as $^2/_6$ (Figure 6.18).

$$\frac{1}{4} + \frac{1}{2} = \frac{2}{6}$$

Figure 6.18

The error

The child has added the two numerators and then the two denominators to calculate the sum.

Why this happens

The child may have been shown the procedure for multiplying fractions (with the same denominator), and has overgeneralised it to addition. They may not know that to add fractions with different denominators, it is often easier to find equivalent fractions to add together. Whatever the specific reason, it is clear that the child is not aware of what the fractions represent and is carrying out the equation without understanding.

Curriculum links

Year 5	Recognise mixed numbers and improper fractions and convert from one form to the other and write mathematical statements > 1 as a mixed number [for example, $^2/_5 + {}^4/_5 = {}^6/_5 = 1\,^1/_5$]
Year 6	Add and subtract fractions with different denominators and mixed numbers, using the concept of equivalent fractions

6.22 Dividing fractions by whole numbers

The teacher has been showing a group of children how to divide fractions by a whole number. When the children are working individually, one child says to the teacher, 'My answer can't be right. One sixth is bigger than one third!'

$$\frac{1}{3} \div 2 = \frac{1}{6}$$

Figure 6.19

The error

The child has achieved a correct answer, but can't believe it is correct because he incorrectly thinks $^1/_6$ is bigger than $^1/_3$.

Why this happens

The child is following a procedure to find the solution, rather than demonstrating any conceptual understanding of the task. He is looking at the denominator of the quotient and thinks that because it is larger than the dividend then it must be wrong. Actually the quotient is smaller but he has not realised this. Experience with cutting up fractions to divide them will support his conceptual understanding.

Curriculum links

Year 5	Multiply proper fractions and mixed numbers by whole numbers, supported by materials and diagrams
Year 6	Divide proper fractions by whole numbers [for example, $^1/_3 \div 2 = ^1/_6$]

6.23 Multiplying and dividing by fractions

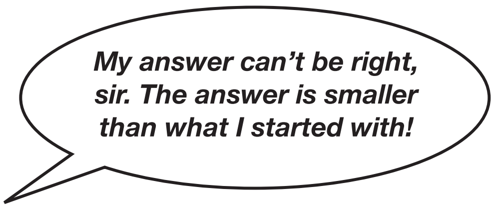


Figure 6.20

The error

The child has achieved a correct answer, but can't believe it is correct because he holds the belief that the result of a multiplication is always a larger number, and the result of a division is always a smaller number.

Why this happens

The child has created an overgeneralisation from previously working with whole numbers. Earlier work in mathematics about multiplication as repeated addition (where the answer always gets bigger) and division as repeated subtraction (where the answer always reduces) can also compound this misconception (although number-line work with fractions can still be used to explain these answers). Additionally, in everyday language, we talk about animals multiplying (increasing in number), which may cause confusion.

Curriculum links

Year 5	Multiply proper fractions and mixed numbers by whole numbers, supported by materials and diagrams
Year 6	Multiply simple pairs of proper fractions, writing the answer in its simplest form [for example, $^1/_4$ x $^1/_2$ = $^1/_8$]

6.24 Reading and ordering decimal numbers

The teacher writes two numbers on the whiteboard (see Figure 6.21). The children are asked to discuss and then identify the largest number. One group claim that 8.35 is the largest number.

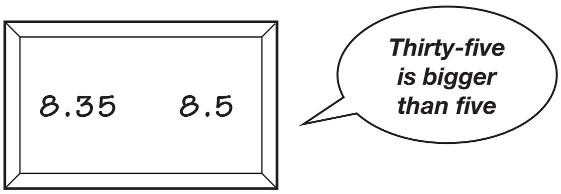


Figure 6.21

The error

When asked why, the group says that 8.35 is larger because 35 is larger than 5 so 8.35 must be larger than 8.5.

Why this happens

The group have read the digits after the decimal point as if they were whole numbers. They may have read the numbers as 'eight point thirty five' and 'eight point five' respectively. They do not understand the relative value of successive groupings of ten in the place-value system for numbers to the right of the decimal point. They need to know that for each place to the right of the decimal point, the numbers are successively smaller by powers of ten. Additionally, some children believe that numbers are larger if there are more decimal digits. Teachers should ensure that children correctly name decimals to help to overcome this difficulty, i.e. 'eight point three five' and 'eight point five'. This type of error may occur because children have been using money as a context for decimals. In money it is quite legitimate to say 'eight, thirty-five'. This is a potential drawback of the use of money to explore the concept of decimals.